Non-Abelian Effects on Wake Potential in Quark-gluon Plasma

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Ref.: Jiang Bing-feng, Li jia-rong, Nucl.Phys.A 832(2010)100

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Outline

- Introduction

- Screening potential: a static and a moving charges

- Screening Potential induced by a fast parton→ the Hard Thermal Loop resummation technique

- Conclusion

Introduction

Jet quenching → a possible signature for the QGP formation→ radiative energy loss.

The experimental dihadron correlation function shows a double peak structure in the away side.(PRL95(2005)152301; PRL97 (2006)052301)

Then, another aspects of the in-medium jet physics, wakes(trail effects) induced by jet, become hot topics.

Mach cone, Cherenkov radiation, large angle gluon radiation...... see a review: hep-ph/0701257.

induced current, induced charges, wake potential
PLB618(2005)123;PRD74(2006)094002.

Wake potential in the linear response theory

A fast parton → a constant velocity and a fixed moving direction, the external(test) charge density

$$\rho^a_{\text{ext}} = 2\pi Q^a \delta(\omega - \mathbf{v} \cdot \mathbf{k}) \qquad (1)$$

The induced color charge density by an external charge

$$\rho^a_{\text{ind}}(\omega, k) = \left(\frac{1}{\epsilon_L(\omega, k)} - 1 \right) \rho^a_{\text{ext}}(\omega, k) \qquad (2)$$

According to Poisson equation, the waked potential

$$\Phi^a(\omega, k) = 4\pi \frac{\rho^a_{\text{ext}}(\omega, k)}{k^2 \epsilon_L(\omega, k)} \qquad (3)$$

ϵ_L is the dielectric function.

$$\Phi^a(\mathbf{r},\mathbf{v},t) = \int \frac{d^3k}{(2\pi)^3} \int \frac{d\omega}{2\pi} \exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)] \cdot \frac{4\pi}{k^2 \epsilon_L(\omega,k)} \cdot 2\pi Q^a \delta(\omega - \mathbf{k}\cdot\mathbf{v})$$

The key point → $\epsilon_L(\omega = \vec{\mathbf{v}} \cdot \vec{\mathbf{k}}, k)$ → the HTL approximation:

$$\epsilon_L^{htl}(\omega,k) = 1 + \frac{m_D^2}{k^2}\left[1 - \frac{\omega}{2k}\left(\ln\left|\frac{\omega+k}{\omega-k}\right| - i\pi\Theta\left(k^2-\omega^2\right)\right)\right],$$

$v = 0 \rightarrow \omega = 0$ the static case, $\Phi^a(r,v,t) = \dfrac{Q\exp[-m_D r]}{r}$

Yukawa potential

$v \neq 0,$ PLB 618(2005)123, PRD 74(2006)094002.

$\epsilon_L(\omega = \vec{\mathbf{v}} \cdot \vec{\mathbf{k}}, k)$ → the HTL result is incomplete

The higher loop contribution → neglected.

In the HTL approximation, $\epsilon_L(\omega = \vec{\mathbf{v}} \cdot \vec{\mathbf{k}}, k)$ of the QGP is similar to the result of the QED abelian plasma, $m_r \rightarrow m_D$

Except the color factors in m_D, no more non-Abelian effects are involved in the dielectric function.

With the HTL resummation approach, → the longitudinal dielectric function → the wakepotential of the moving test charge.

The resumed dielectric function and the wake potential

- In Coulomb gauge

$$\epsilon_L(\omega, k) = 1 - \frac{\Pi_{00}(K)}{k^2}$$

- The gluon self-energy

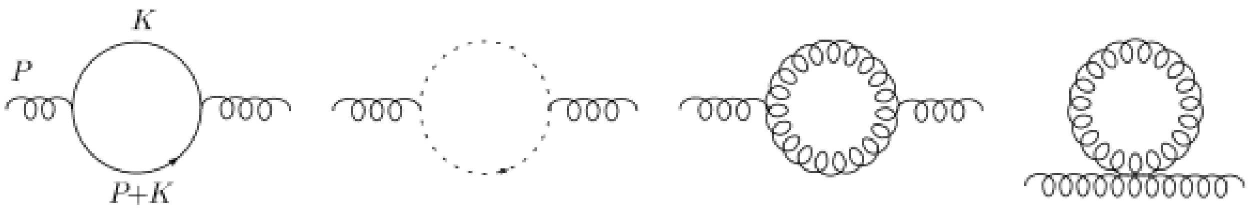


Fig 2. One-loop gluon self-energy

- In the HTL approximation

$$\Pi_L^{htl}(K) = \Pi_{00}^{htl}(K) = -m_D^2\left[1 - \frac{\omega}{2k}\ln\left(\frac{\omega+k}{\omega-k}\right)\right],$$

$$\Pi_T^{htl}(K) = \frac{m_D^2}{2}\frac{\omega^2}{k^2}\left[1 - \left(1 - \frac{\omega^2}{k^2}\right)\frac{\omega}{2k}\ln\left(\frac{\omega+k}{\omega-k}\right)\right],$$

In the HTL resummation approximation

- The integrals over loop momentom→“hard” (~T) and “soft”(~gT) momentom. Braaten & Pisarsiki, NPB 337&339, PRL 63&64.

- Results from integral over the hard momentum at high temperature $\Pi^{h}_{\mu\nu}(P)$ → the HTL self-energy $\Pi^{htl}_{\mu\nu}(P)$

- In the case of the integrals over soft momentum, effective propagators and effective vertices are involved and the calculation are much complicated.

- Assuming the external gluon line is hard.The soft momentum contribution mainly comes from diagrams

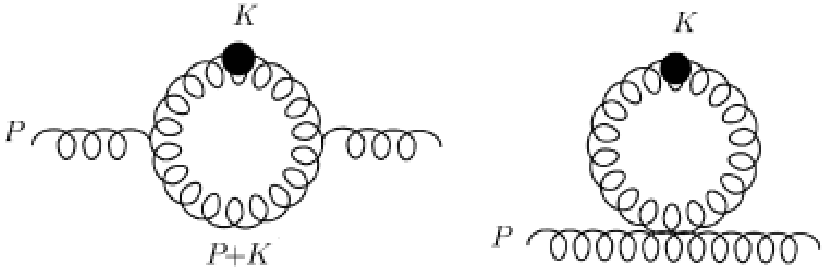


Fig 3. The effective gluon self-energy

- The self-polarization of gluon →no countpart in Abelian plasma →non-Abelian characteristic of the QGP

- The dotted line →the effective gluon propagator $D^{*\mu\nu}$ is defined by the Schwinger-Dyson equation in the Coulomb gauge. Its longitudinal and transverse components are

$$D_{00}(K) = \Delta_L(K), \quad \Delta_L(K) = \frac{i}{k^2};$$

$$D_{0i}(K) = D_{i0}(K) = 0;$$

$$D_{ij}(K) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\Delta_T(K), \quad \Delta_T(K) = \frac{i}{K^2}.$$

$$\Delta_L^*(K) = \frac{i}{k^2 - \Pi_L^{htl}(K)}, \qquad \Delta_T^*(K) = \frac{i}{K^2 - \Pi_T^{htl}(K)}.$$

- According to the effective gluon propagator, the effective gluon self-energy

$$\Pi_{\mu\nu}^{*1}(K) = \frac{1}{2} i \int \frac{d^4P}{(2\pi)^4} \Gamma_{\sigma\mu\rho}(-K-P, K, P) D_{\rho\rho'}^*(P) \times \Gamma_{\rho'\nu\sigma}(-P, -K, P+K) D_{\sigma\sigma'}(P+K)$$

$$\Pi_{\mu\nu}^{*2}(K) = \frac{1}{2} i \int \frac{d^4P}{(2\pi)^4} \Gamma_{\mu\nu\rho\sigma}(K, -K, P, -P) D_{\rho\sigma}^*(P)$$

$$\Pi_{\mu\nu}^*(K) = \Pi_{\mu\nu}^{*1}(K) + \Pi_{\mu\nu}^{*2}(K)$$

$$\Pi_{00}^*(K) = \Pi_{00}^{*1}(K) + \Pi_{00}^{*2}(K)$$

- The resummed gluon self-energy and dielectric function

$$\Pi_{00}(K) \approx \Pi_{00}^{htl}(K) + \Pi_{00}^*(K), \quad \epsilon_L(\omega, k) = 1 - \frac{\Pi_{00}(K)}{k^2}$$

• Assuming the fast parton → z, in the cylindrical coordination

$$\mathbf{k} = (\kappa\cos\phi, \kappa\sin\phi, k_z), \mathbf{r} = (\rho, 0, z)$$

$$\Phi^a(\rho, z, t) = \frac{Q^a}{\pi v}\int_0^\infty d\kappa\, \kappa J_0(\kappa\rho)\int_{-\infty}^{\infty} d\omega\, \frac{1}{k^2\Delta(\omega, k)} \times \left[\cos\left\{\omega\left(\frac{z}{v} - t\right)\right\}\cdot \mathrm{Re}\,\epsilon_L + \sin\left\{\omega\left(\frac{z}{v} - t\right)\right\}\cdot \mathrm{Im}\,\epsilon_L\right]$$

where J_0 is the Bessel function, $k = \sqrt{\kappa^2 + \omega^2/v^2}$ and $\Delta = (\mathrm{Re}\,\epsilon_L)^2 + (\mathrm{Im}\,\epsilon_L)^2$

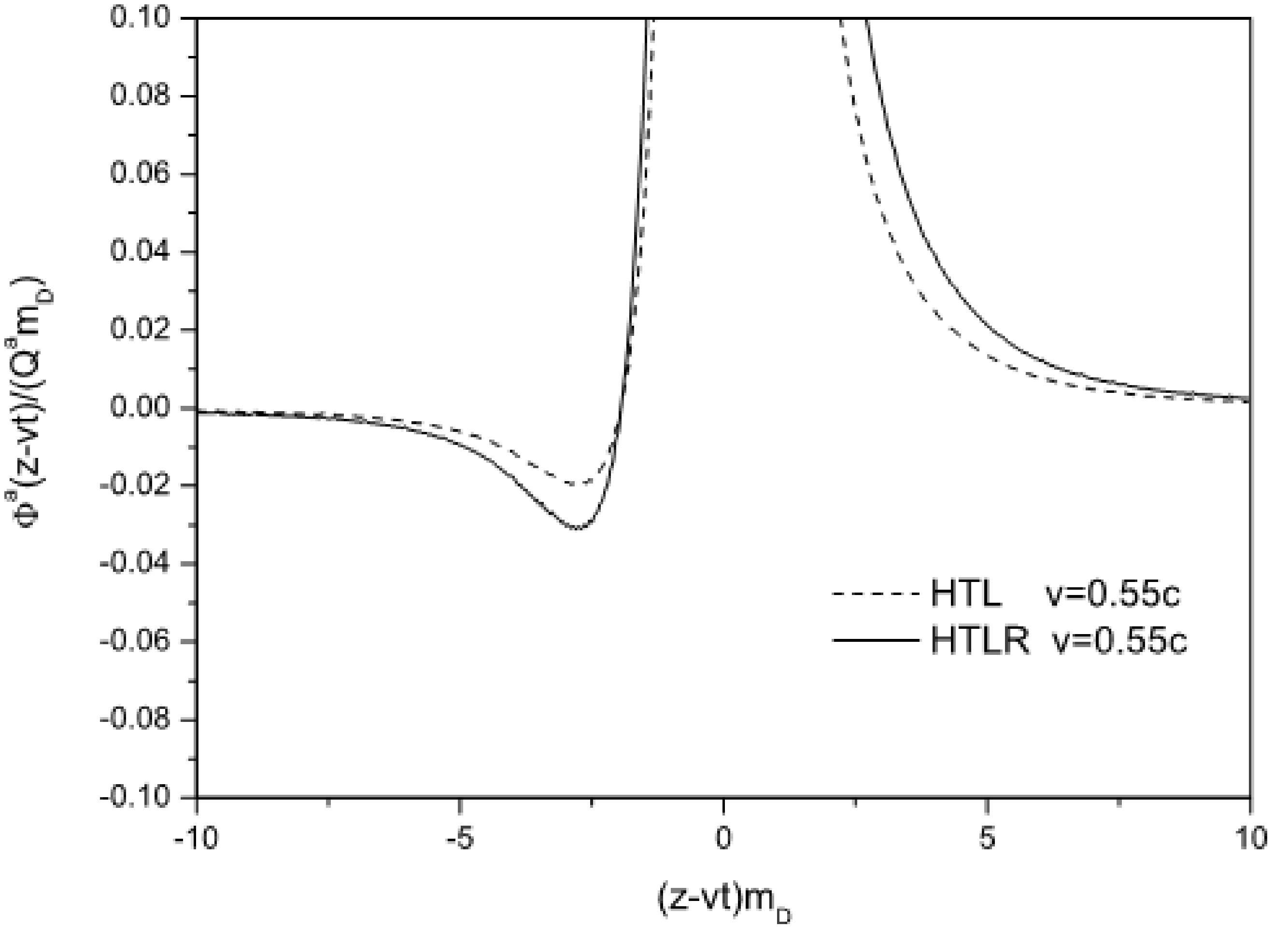
• Focusing on the screening potential parallel to the moving direction of the fast parton, $\mathbf{r} \parallel \mathbf{v}$ and $\rho = 0$

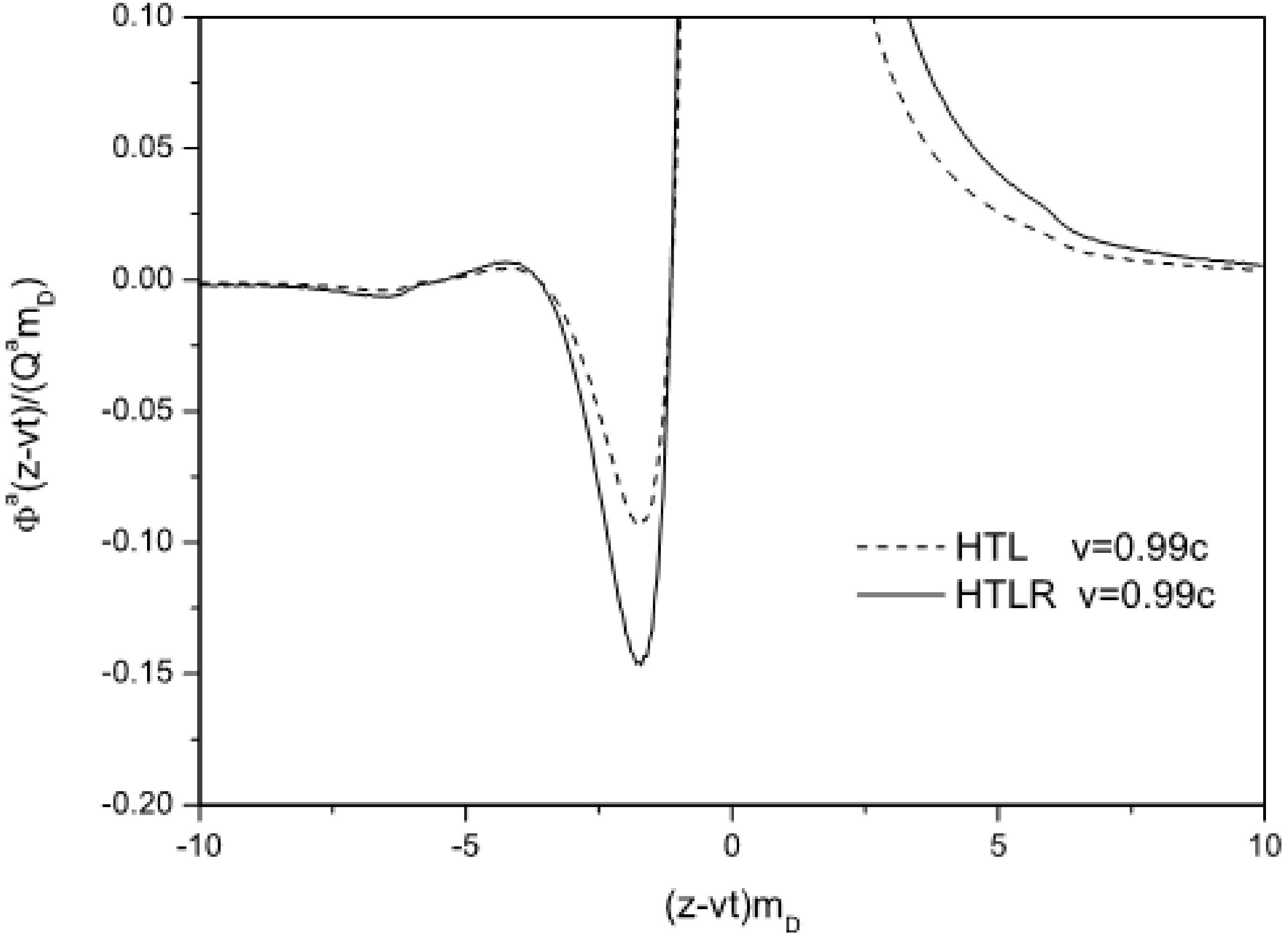
$$\frac{\Phi^a_{\parallel}(\mathbf{z}, \mathbf{v}, t)}{Q^a m_D} = \frac{1}{\pi}\int_0^\infty dk\int_{-1}^{1} dx\left\{\frac{\mathrm{Re}\,\epsilon_L}{\Delta}\cos(kx|\mathbf{z} - \mathbf{v}t|m_D) + \frac{\mathrm{Im}\,\epsilon_L}{\Delta}\sin(kx|\mathbf{z} - \mathbf{v}t|m_D)\right\}$$

$$= \Phi_1 + \Phi_2.$$

Numerical results and discussion

- In numerical calculation, $g = 0.1, N_f = 2, C_A = 3$
- For comparision, HTL & HTL resummation





0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-10
-5
0
5
10
$\Phi^a(z\text{-}vt)/(Q^a m_D)$
$(z\text{-}vt)m_D$
HTL v=0.99c
HTLR v=0.99c

- Screening potential in forward-backward direction is anisotropic.
  - In the forward direction→ Yukawa-like potential
  - In the backward direction→ speed-dependent,

    $v = 0.55c$ Lennard-Jones-like potential with a negative mimimum → a short repulsive and a long range attractive inteaction.

    $v = 0.99c$ → Showing an obvious oscillation

For comparision of results → HTL and HTL resummation

- Resummation calculation enhances anisotropy of the sceening potential.
  - In the forward direction→ screening reduced in resummation
  - In the backward direction→ the negative mimimum→ deeper

How to understand these properties ?

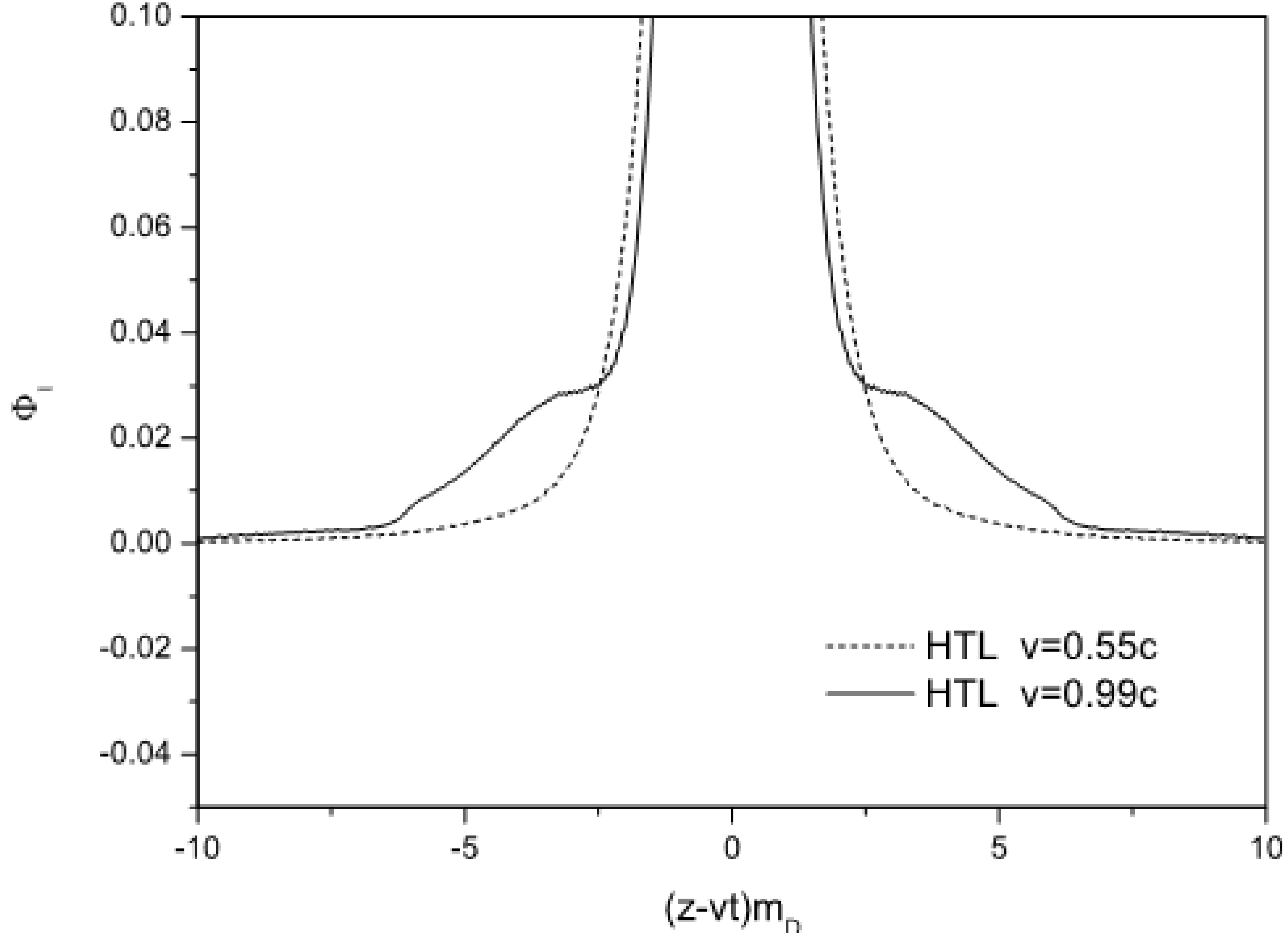
$$\frac{\Phi_{\parallel}^{a}(\mathbf{z},\mathbf{v},t)}{Q^{a} m_D} = \frac{1}{\pi}\int_0^{\infty} dk \int_{-1}^{1} dx \left\{ \frac{\mathrm{Re}\,\epsilon_L}{\Delta}\cos(kx|\mathbf{z}-\mathbf{v}t|m_D) + \frac{\mathrm{Im}\,\epsilon_L}{\Delta}\sin(kx|\mathbf{z}-\mathbf{v}t|m_D) \right\}$$
$$= \Phi_1 + \Phi_2.$$

- The HTL dielectric function

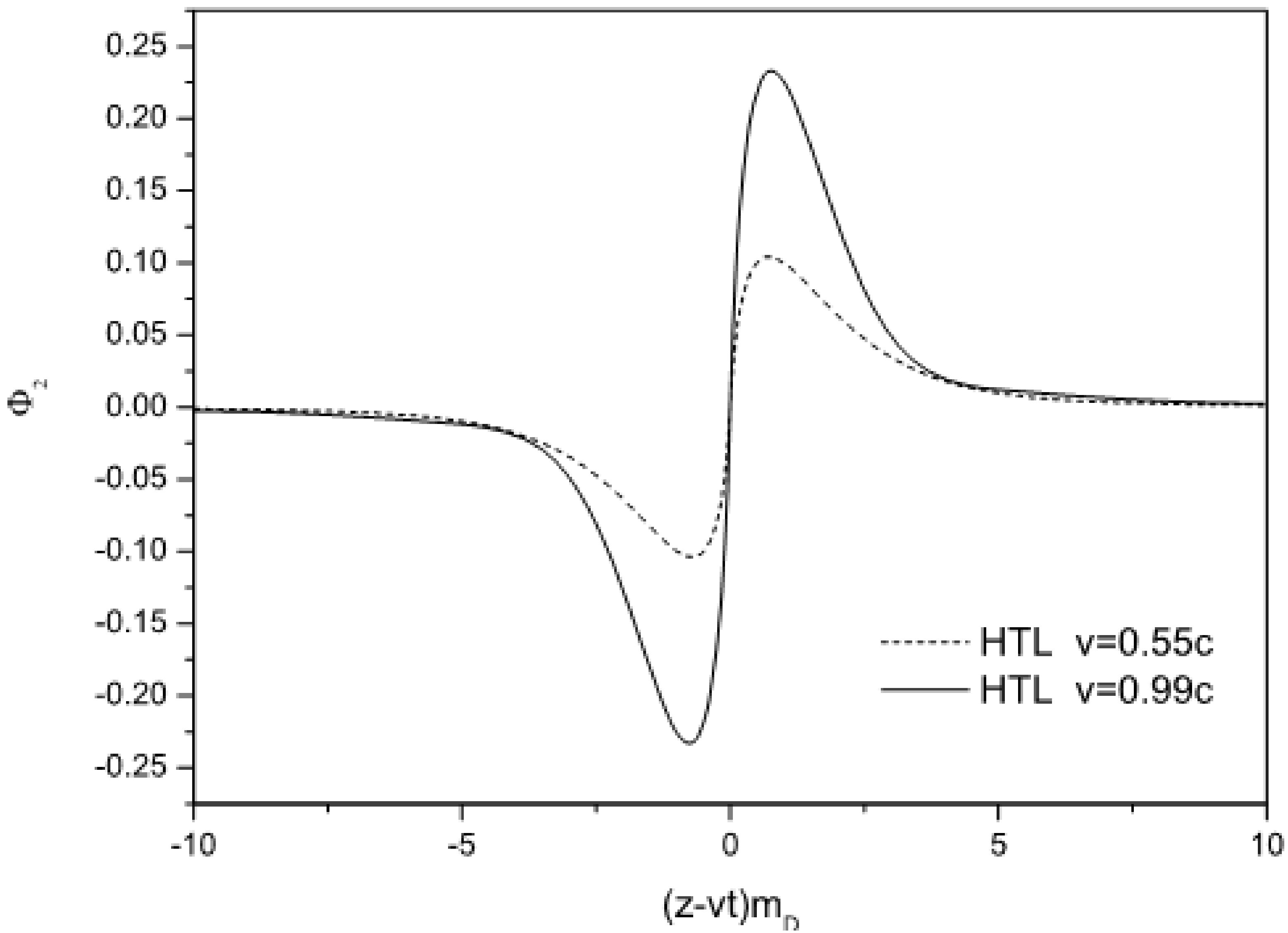
$$\mathrm{Re}\,\epsilon_L^{htl}(\omega,k) = 1 + \frac{m_D^2}{k^2}\left[1 - \frac{\omega}{2k}\ln\left|\frac{\omega+k}{\omega-k}\right|\right],$$
$$\mathrm{Im}\,\epsilon_L^{htl}(\omega,k) = \frac{m_D^2}{k^2}\cdot\frac{\omega}{2k}\pi\,\Theta(k^2-\omega^2).$$

- Symmetric properties about ω

$$\mathrm{Re}\,\epsilon_L^{htl}(\omega,k) = \mathrm{Re}\,\epsilon_L^{htl}(-\omega,k),$$
$$\mathrm{Im}\,\epsilon_L^{htl}(\omega,k) = -\mathrm{Im}\,\epsilon_L^{htl}(-\omega,k).$$



0.10
0.08
0.06
0.04
0.02
0.00
-0.02
-0.04
Φ_i
HTL v=0.55c
HTL v=0.99c
-10
-5
0
5
10
(z-vt)m_D



0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
Φ2
HTL v=0.55c
HTL v=0.99c
-10
-5
0
5
10
(z-vt)mD

- The antisymmetric property of $\mathrm{Im}\,\epsilon_L(\omega = kvx, k)$ about ω results in the anisotropy of the wake potential
- $v\uparrow \rightarrow \mathrm{Im}\in(\omega = kvx, k)\uparrow \rightarrow$ the absolute value of Φ_2 → anisotropy enhancement of the wake potential
- The wake structures, ie. the oscillatory or the Lennard-Jones potential are also determined by $\mathrm{Im}\,\epsilon_L(\omega = kvx, k)$

  $\mathrm{Im}\,\epsilon_L(\omega = kvx, k)$ influences Φ_1 through Δ . When it is small,its contribution to Φ_1 inappreciable → the Lennard - Jones potential. While it is large enough, it changes Φ_1 remarkably → oscillatory potential
- The resummation calculation give a correction to the imagiary part of the dielectric function→ result in the enhancement of the anisotropy of the screening potential.

Jiang Bing-feng& Li Jia-rong, CTP49(2008)1567, Zhang Xiao-fei& Li Jia-rong PRC52(1995)964, Zheng Xiao-ping & Li Jia-rong, PLB 409(1997)45.

Conclusion

- The screening potential of a moving test charge→ anisotropy in forward-backward directrion

- The anisotropic behavior, wake structure, ie. the oscillatory potential or the Lennard-Jones potential are attributed to the imaginary part of the dielectric function.

- The HTL resummation calculation→ corrective contribution to imaginary part of the dielectric function→ enhance anisotropy