D波重味介子的强衰变和电磁衰变

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10.04.18

重味介子和轻介子的耦合常数

重味介子的电磁耦合常数

衰变宽度

- 近年来，实验上发现一系列难以纳入夸克模型的强子态，如近阈的介子$X(3872)$、$Y(4260)$、$Z^{+}(4430)$等。它们可能是重味介子间通过交换轻介子形成的分子态，对重味介子和轻介子之间强作用的研究也许有助于理解这些态。
- 对D波重味介子衰变模式的研究有助于实验上确认这些介子。

重夸克极限下的重味介子

在重夸克极限下($m_Q \to \infty$)，重味介子中轻自由度的角动量是好量子数。此时介子谱由一系列具有确定宇称和轻自由度自旋的二重态组成。比如：

- H: $j_l^P = \frac{1}{2}^-$，$(0^-, 1^-)$，s波
- S: $j_l^P = \frac{1}{2}^+$，$(0^+, 1^+)$，p波
  T: $j_l^P = \frac{3}{2}^+$，$(1^+, 2^+)$，p波
- M: $j_l^P = \frac{3}{2}^-$，$(1^-, 2^-)$，d波
  N: $j_l^P = \frac{5}{2}^-$，$(2^-, 3^-)$，d波

试探流

Y.-B. Dai *et al*. PLB390, 350 (1997)给出了重夸克有效理论中重味介子试探流的一般形式，由此可写出相应的试探流如：

$$
\begin{aligned}
J^{\dagger}_{H_0} &= \sqrt{\frac{1}{2}}\bar{h}_v \gamma_5 q \, , \\
J^{\dagger\alpha}_{H_1} &= \sqrt{\frac{1}{2}}\bar{h}_v \gamma_t^{\alpha} q \, , \\
J^{\dagger}_{S_0} &= \sqrt{\frac{1}{2}}\bar{h}_v q \, , \\
J^{\dagger\alpha}_{S_1} &= \sqrt{\frac{1}{2}}\bar{h}_v \gamma_5 \gamma_t^{\alpha} q \, , \\
J^{\dagger\alpha}_{M_1} &= \sqrt{\frac{3}{4}}\bar{h}_v (-i) \left\{ \mathcal{D}_t^{\alpha} - \frac{1}{3}\gamma_t^{\alpha} \hat{\mathcal{D}}_t \right\} q \, , \\
J^{\dagger\alpha_1\alpha_2}_{M_2} &= \sqrt{\frac{1}{8}}\bar{h}_v (-i)\gamma_5 \left\{ \gamma_t^{\alpha_1} \mathcal{D}_t^{\alpha_2} + \gamma_t^{\alpha_2} \mathcal{D}_t^{\alpha_1} - \frac{2}{3} g_t^{\alpha_1\alpha_2} \hat{\mathcal{D}}_t \right\} q \, . \qquad (1)
\end{aligned}
$$

在$m_Q \to \infty$ 极限下，这些试探流满足：

$$\begin{aligned}
\langle 0|J^{\alpha_1\cdots\alpha_j}_{j,P,j_\ell}(0)|j',P',j'_\ell\rangle &= f_{Pj_\ell}\delta_{jj'}\delta_{PP'}\delta_{j_\ell j'_\ell}\eta^{\alpha_1\cdots\alpha_j}\ , \\
i\,\langle 0|T\left(J^{\alpha_1\cdots\alpha_j}_{j,P,j_\ell}(x)J^{\dagger\beta_1\cdots\beta_{j'}}_{j',P',j'_\ell}(0)\right)|0\rangle &= \delta_{jj'}\delta_{PP'}\delta_{j_\ell j'_\ell}(-1)^j\,\mathcal{S}\,g_t^{\alpha_1\beta_1}\cdots g_t^{\alpha_j\beta_j} \\
&\quad \times\int dt\delta(x-vt)\,\Pi_{P,j_\ell}(x)\,,
\end{aligned}$$

其中$\eta^{\alpha_1\cdots\alpha_j}$为极化张量，$v$为重夸克速度，$\mathcal{S}$ 表示对指标$(\alpha_1\cdots\alpha_j)$ 和$(\beta_1\cdots\beta_{j'})$的对称化和无迹操作。
上式表明在重夸克极限下，不同j、P、j_ℓ的流在夸克层次和介子层次上都没有混合，它们体现了重夸克对称性。

关联函数

以$M_2 \to H_1 + \pi$为例，其衰变振幅为：

$$\mathcal{M}(M_2 \to H_1 + \pi) \;=\; I\eta_{\alpha_1\alpha_2}[\epsilon_t^{*\alpha_1} q_t^{\alpha_2} - \frac{1}{3} g_t^{\alpha_1\alpha_2}(\epsilon^* \cdot q_t)] g^{p1}_{M_2 H_1 \pi}\,, \quad (2)$$

其中η和ϵ^*分别为M_2和H_1的极化矢量；q为末态π介子的动量，$q^2 = m_\pi^2$，$q_t^\mu \equiv q^\mu - (q\cdot v)v^\mu$；$\mathrm{I} = 1, 1/\sqrt{2}$分别为带电和中性$\pi$介子的同位旋因子。

为得出关于$g^{p1}_{M_2H_1\pi}$的求和规则，考虑如下关联函数：

$$\begin{aligned}&\int d^4x e^{-ik\cdot x}\langle\pi(q)|T\{J^{\beta}_{1,-,\frac{1}{2}}(0)J^{\dagger\alpha_1\alpha_2}_{2,-,\frac{3}{2}}(x)\}|0\rangle\\ =\;& I\left[\frac{1}{2}(g_t^{\alpha_1\beta}q_t^{\alpha_2}+g_t^{\alpha_2\beta}q_t^{\alpha_1})-\frac{1}{3}g_t^{\alpha_1\alpha_2}q_t^{\beta}\right]G^{p1}_{M_2H_1\pi}(\omega,\omega'),\quad (3)\end{aligned}$$

其中$\omega\equiv 2v\cdot k$，$\omega'\equiv 2v\cdot(k-q)$。

强子层次的关联函数

在强子层次上，$G_{M_2H_1\pi}(\omega,\omega')$有以下极点项：

$$G^{p1}_{M_2H_1\pi}(\omega,\omega') = \frac{f_{-,1/2}f_{-,3/2}g^{p1}_{M_2H_1\pi}}{(2\bar{\Lambda}_{-,1/2}-\omega')(2\bar{\Lambda}_{-,3/2}-\omega)} + \frac{c}{2\bar{\Lambda}_{-,1/2}-\omega'} + \frac{c'}{2\bar{\Lambda}_{-,3/2}-\omega} + \cdots, \quad (4)$$

其中$\bar{\Lambda}_{-,1/2}\equiv m_H - m_Q$，$\bar{\Lambda}_{-,3/2}\equiv m_M - m_Q$。$f_{-,1/2}$等为插入流和重介子的重叠振幅。单极点项在双Borel变换后消去。

夸克层次的关联函数

利用重夸克极限下的夸克传播子

$$\langle 0|T\{h_v(0)\bar{h}_v(x)\}|0\rangle = \frac{1+\hat{v}}{2}\int dt\delta^4(-x-vt)\,, \qquad (5)$$

可将关联函数在夸克层次上写成

$$-\frac{i}{4}\int dxe^{-ik\cdot x}\int_0^\infty dt\delta(-x-vt)$$
$$\mathrm{Tr}\left\{\gamma_t^\beta\frac{1+\hat{v}}{2}\gamma_5\left[\gamma_t^{\alpha_1}\mathcal{D}_t^{\alpha_2}+\gamma_t^{\alpha_2}\mathcal{D}_t^{\alpha_1}-\frac{2}{3}g_t^{\alpha_1\alpha_2}\hat{\mathcal{D}}_t\right]\langle\pi(q)|q(x)\bar{q}(0)|0\rangle\right\}, \quad (6)$$

利用π的光锥波函数即可完成夸克层次上$G_{M_2H_1\pi}(\omega,\omega')$的计算。

$g_{M_2 H_1 \pi}$

对强子层次和夸克层次的表达式作双Borel变换得出$g_{M_2 H_1 \pi}$的求和规则：

$$
\begin{aligned}
& g^{p1}_{M_2 H_1 \pi} f_{-,\frac{1}{2}} f_{-,\frac{3}{2}} e^{-\frac{\bar{\Lambda}_{-,3/2}+\bar{\Lambda}_{-,1/2}}{T}} \\
&= -\frac{1}{48} f_\pi \Big\{ -12 \left[\phi'_\pi(\bar{u}_0) - (u\phi_\pi)'(\bar{u}_0)\right] T^2 f_1(\frac{\omega_c}{T}) \\
&\quad - \frac{4m_\pi^2}{m_u + m_d} \left[6\mathcal{T}^{[1,0]}(u_0) + 6\phi_p(\bar{u}_0) - 6(u\phi_p)(\bar{u}_0) + \phi_\sigma(\bar{u}_0) \right] T f_0(\frac{\omega_c}{T}) \\
&\quad + 3m_\pi^2 \Big[\mathbb{A}'(\bar{u}_0) - (u\mathbb{A})'(\bar{u}_0) + 8\mathbb{B}^{[2]}(\bar{u}_0) - 8\mathbb{B}^{[1]}(\bar{u}_0) + 8(u\mathbb{B})^{[1]}(\bar{u}_0) \\
&\quad - 16\mathcal{V}_\perp^{[0,0]}(u_0) - 16\mathcal{A}_\parallel^{[0,0]}(u_0) \Big] \Big\} , \qquad (7)
\end{aligned}
$$

其中$f_n(x) = 1 - e^{-x} \sum_{k=0}^{n} \frac{x^k}{k!}$为连续谱减除因子，$\omega_c$为连续谱的阈；$u_0 = \frac{T_1}{T_1+T_2}$，$T = \frac{T_1 T_2}{T_1+T_2}$，$T_1$和$T_2$为Borel参数；$\bar{u}_0 = 1 - u_0$.

一些函数

以上求和规则中的一些函数定义如下：

$$
\begin{aligned}
\mathcal{F}^{[n]}(\bar{u}_0) &\equiv \int_0^{\bar{u}_0}\cdots\int_0^{x_3}\int_0^{x_2}\mathcal{F}(x_1)dx_1dx_2\cdots dx_n,\\
\mathcal{F}^{[0,0]}(u_0) &\equiv \int_0^{u_0}\int_{u_0-\alpha_2}^{1-\alpha_2}\frac{\mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3)}{\alpha_3}d\alpha_3d\alpha_2,\\
\mathcal{F}^{[1,0]}(u_0) &\equiv \int_0^{u_0}\frac{\mathcal{F}(1-u_0,\alpha_2,u_0-\alpha_2)}{u_0-\alpha_2}d\alpha_2\\
&\quad -\int_0^{1-u_0}\frac{\mathcal{F}(u_0,1-u_0-\alpha_3,\alpha_3)}{\alpha_3}d\alpha_3,\\
\mathcal{F}^{[2,0]}(u_0) &\equiv \int_0^{u_0}d\alpha_2\left.\frac{\partial[\mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3)]/\partial\alpha_2}{\alpha_3}\right|_{\alpha_3=u_0-\alpha_2}\\
&\quad -\int_0^{1-u_0}d\alpha_3\left.\frac{\partial[\mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3)]/\partial\alpha_2}{\alpha_3}\right|_{\alpha_2=u_0},\\
\mathcal{F}^{[-1,0]}(u_0) &\equiv \int_0^{u_0}\int_0^{u_0-\alpha_2}\mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3)d\alpha_3d\alpha_2\\
&\quad +\int_0^{u_0}\int_{u_0-\alpha_2}^{1-\alpha_2}\frac{(u_0-\alpha_2)\mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3)}{\alpha_3}d\alpha_3d\alpha_2.
\end{aligned}
$$

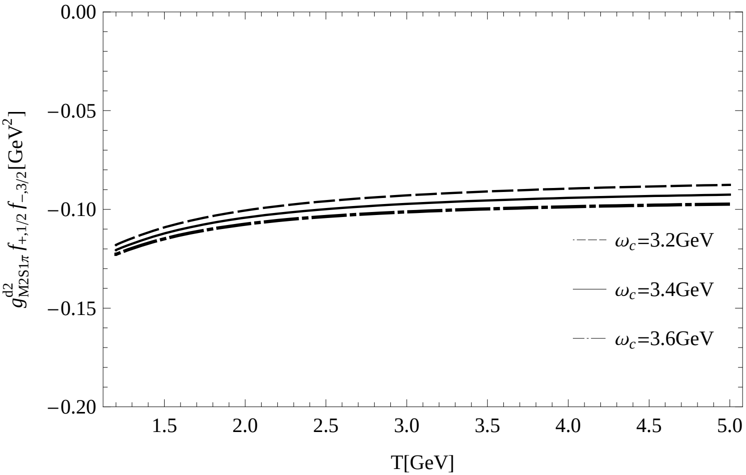

Figure: The sum rule for $g^{d2}_{M_2S_1\pi} f_{+,1/2} f_{-,3/2}$ with $\omega_c = 3.2, 3.4, 3.6$ GeV and the working interval $3.0 < T < 4.0$ GeV.

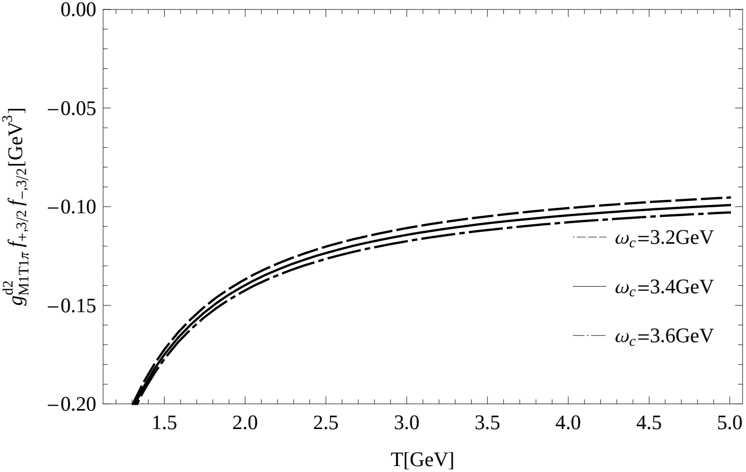

Figure: The sum rule for $g^{d2}_{M_1 T_1 \pi} f_{+,3/2} f_{-,3/2}$ with $\omega_c = 3.2, 3.4, 3.6$ GeV and the working interval $3.0 < T < 4.0$ GeV.

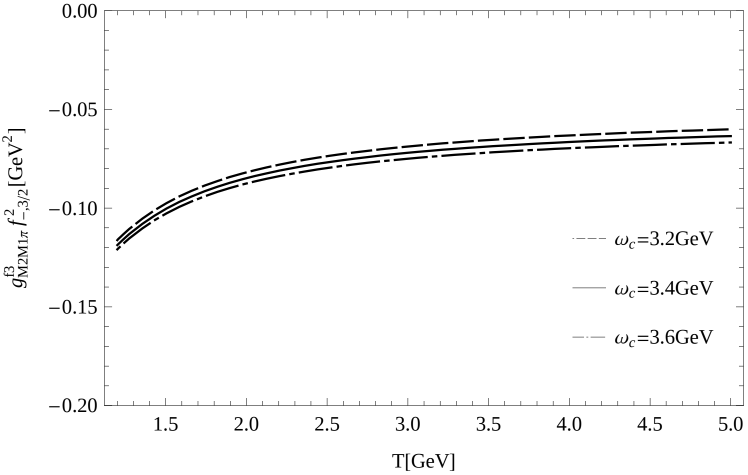

Figure: The sum rule for $g^{f3}_{M_2 M_1 \pi} f^2_{-,3/2}$ with $\omega_c = 3.2, 3.4, 3.6$ GeV and the working interval $3.0 < T < 4.0$ GeV.

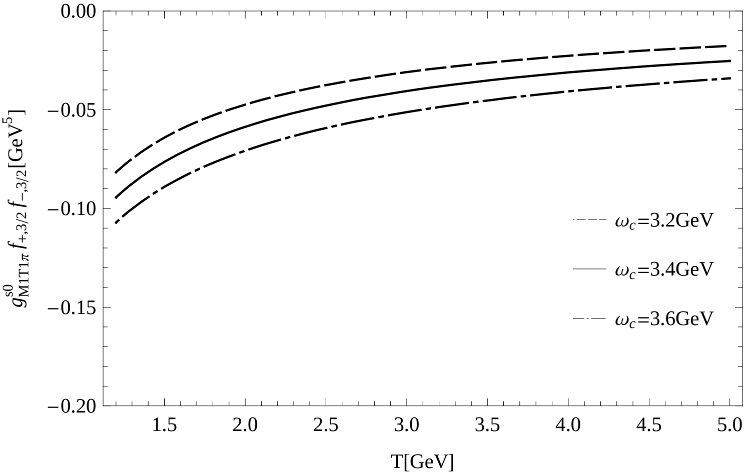

Figure: The sum rule for $g^{s0}_{M_1 T_1 \pi} f_{+,3/2} f_{-,3/2}$ with $\omega_c = 3.2, 3.4, 3.6$ GeV. There is no working interval for the Borel parameter T.

输入参数：$\bar{\Lambda}$和f

$$
\begin{aligned}
\bar{\Lambda}_{-,1/2} &= 0.50\ \text{GeV}, \quad f_{-,1/2} = 0.25 \pm 0.04\ \text{GeV}^{3/2}, \\
\bar{\Lambda}_{+,1/2} &= 0.85\ \text{GeV}, \quad f_{+,1/2} = 0.36 \pm 0.10\ \text{GeV}^{3/2}, \\
\bar{\Lambda}_{+,3/2} &= 0.95\ \text{GeV}, \quad f_{+,3/2} = 0.26 \pm 0.06\ \text{GeV}^{5/2}, \\
\bar{\Lambda}_{-,3/2} &= 1.42\ \text{GeV}, \quad f_{+,1/2} = 0.39 \pm 0.03\ \text{GeV}^{3/2}.
\end{aligned}
$$

M. Neubert, Phys. Rev. **D45**, 2451(1992); S. L. Zhu and Y. B. Dai, Mod. Phys. Lett. **A14**, 2367 (1999); W. Wei, X. Liu, and S. L. Zhu, Phys. Rev. **D75**, 014013 (2007).

输入参数：π介子光锥波函数中的参数

a_2	η_3	ω_3	η_4	ω_4	h_{00}
0.25	0.015	-1.5	10	0.2	-3.33
v_{00}	a_{10}	v_{10}	h_{01}	h_{10}	
-3.33	5.14	5.25	3.46	7.03	

P. Ball, JHEP **9901**, 010 (1999); P. Ball, V. M. Braun, and A. Lenz, JHEP **0605**, 004 (2006).

π耦合常数的数值结果：

	$g^{p1*}_{M_2H_1\pi}$	$g^{d2}_{M_2S_1\pi}$	$g^{s0*}_{M_1T_1\pi}$
g_c	1.13	−0.68	−0.40
g	0.78 ∼ 1.65	−1.07 ∼ −0.47	−0.83 ∼ −0.15
	$g^{d2}_{M_1T_1\pi}$	$g^{p1*}_{M_2M_1\pi}$	$g^{f3}_{M_2M_1\pi}$
g_c	−1.09	0.04	−0.46
g	−1.67 ∼ −0.75	0.03 ∼ 0.05	−0.62 ∼ −0.34

Table: π耦合常数（单位$[\mathrm{GeV}]^{-j}$，j为末态π介子的轨道角动量。）g_c对应于重叠振幅、连续谱阈值以及Borel变量在其工作区的中间值：$\omega_c = 3.4$ GeV，$T = 3.5$ GeV。星号表示相应的求和规则没有稳定的工作区。

$$
\begin{aligned}
g^{p1}_{M_2H_1\pi} &= \frac{\sqrt{6}}{2} g^{p1}_{M_1H_0\pi} = \sqrt{6} g^{p1}_{M_1H_1\pi} \,, \\
g^{d2}_{M_2S_1\pi} &= -\frac{\sqrt{6}}{2} g^{d2}_{M_1S_1\pi} = -g^{d2}_{M_2S_0\pi} \,, \\
g^{s0}_{M_1T_1\pi} &= g^{s0}_{M_2T_2\pi} \,, \\
g^{d2}_{M_1T_1\pi} &= -\sqrt{6} g^{d2}_{M_1T_2\pi} = -\sqrt{6} g^{d2}_{M_2T_1\pi} = -\frac{3}{2} g^{d2}_{M_2T_2\pi} \,, \\
g^{p1}_{M_2M_1\pi} &= -\frac{\sqrt{6}}{10} g^{p1}_{M_1M_1\pi} = \frac{\sqrt{6}}{3} g^{p1}_{M_2M_2\pi} \,, \\
g^{f3}_{M_2M_1\pi} &= 2\sqrt{6} g^{f3}_{M_2M_2\pi} \,.
\end{aligned} \tag{8}
$$

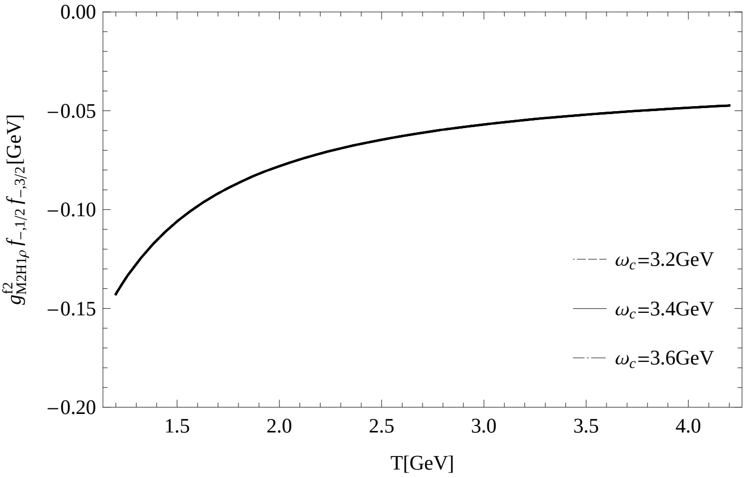

Figure: The sum rule for $g^{f2}_{M_2H_1\rho}f_{-,1/2}f_{-,3/2}$ with $\omega_c = 3.2, 3.4, 3.6$ GeV and the working interval $2.5 < T < 3.0$ GeV.

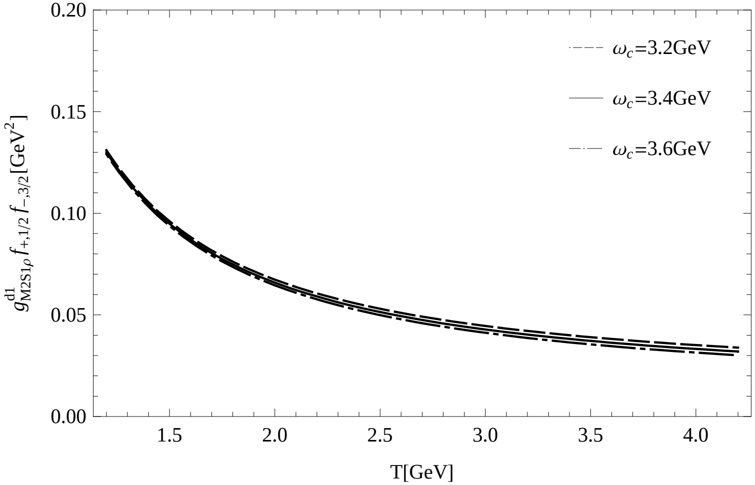

Figure: The sum rule for $g^{d1}_{M_2 S_1 \rho} f_{+,1/2} f_{-,3/2}$ with $\omega_c = 3.2, 3.4, 3.6$ GeV and the working interval $2.5 < T < 3.0$ GeV.

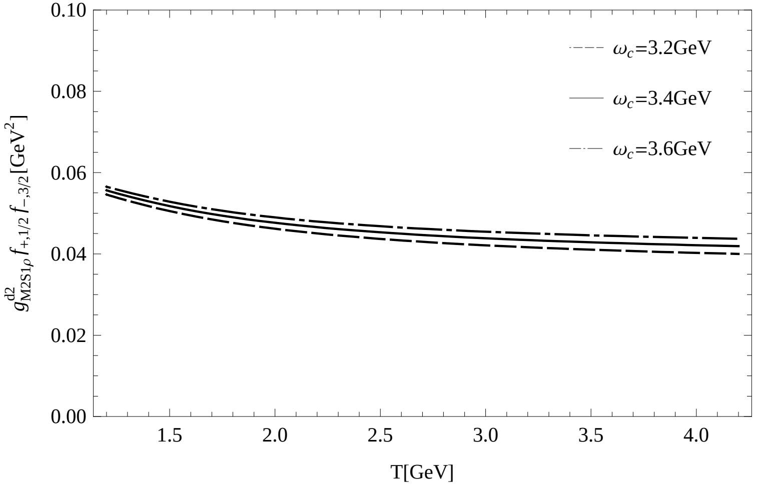

Figure: The sum rule for $g^{d2}_{M_2 S_1 \rho} f_{+,1/2} f_{-,3/2}$ with $\omega_c = 3.2, 3.4, 3.6$ GeV and the working interval $2.5 < T < 3.0$ GeV.

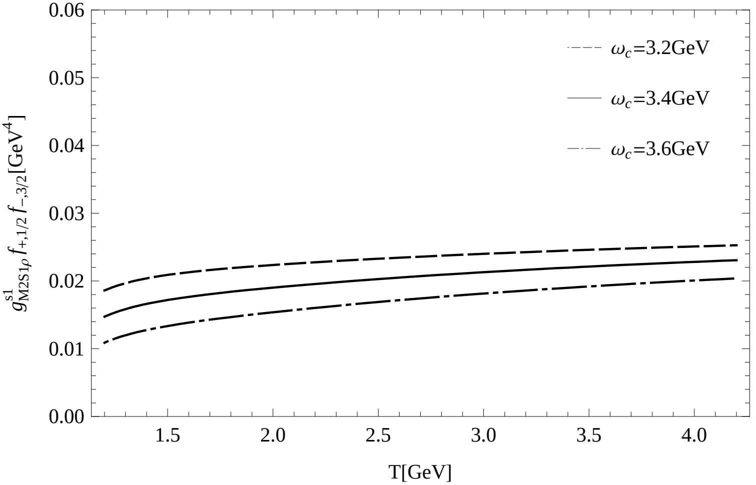

Figure: The sum rule for $g^{s1}_{M_2 S_1 \rho} f_{+,1/2} f_{-,3/2}$ with $\omega_c = 3.2, 3.4, 3.6$ GeV. There is no working interval for the Borel parameter T.

输入参数：ρ介子光锥波函数中的参数

$f_\rho^\parallel$[MeV]	$f_\rho^\perp$[MeV]	$a_2^\parallel$	$a_2^\perp$	$\zeta_{3\rho}^\parallel$	$\tilde{\omega}_{3\rho}^\parallel$
216(3)	165(9)	0.15(7)	0.14(6)	0.030(10)	$-$0.09(3)
$\omega_{3\rho}^\parallel$	$\omega_{3\rho}^\perp$	$\zeta_4^\parallel$	$\tilde{\omega}_4^\parallel$	$\zeta_4^\perp$	$\tilde{\zeta}_4^\perp$
0.15(5)	0.55(25)	0.07(3)	$-$0.03(1)	$-$0.03(5)	$-$0.08(5)

P. Ball and G. W. Jones, JHEP **0703**, 069 (2007); P. Ball, V. M. Braun and A. Lenz, JHEP **0708**, 090 (2007).

ρ耦合常数的数值结果：

	$g^{p1*}_{M_2H_1\rho}$	$g^{p2*}_{M_2H_1\rho}$	$g^{f2}_{M_2H_1\rho}$	$g^{s1*}_{M_2S_1\rho}$	$g^{d1}_{M_2S_1\rho}$	$g^{d2}_{M_2S_1\rho}$
g_c	-0.10	-0.26	-0.62	0.16	0.32	0.30
g	$-0.20\sim-0.04$	$-0.40\sim-0.16$	$-0.93\sim-0.41$	$0.09\sim0.27$	$0.21\sim0.53$	$0.21\sim0.47$
	$g^{p1*}_{M_2H_1\omega}$	$g^{p2*}_{M_2H_1\omega}$	$g^{f2}_{M_2H_1\omega}$	$g^{s1*}_{M_2S_1\omega}$	$g^{d1}_{M_2S_1\omega}$	$g^{d2}_{M_2S_1\omega}$
g_c	-0.08	-0.24	-0.56	0.14	0.32	0.27
g	$-0.13\sim-0.04$	$-0.34\sim-0.16$	$-0.79\sim-0.41$	$0.08\sim0.25$	$0.21\sim0.53$	$0.19\sim0.43$

Table: ρ/ω耦合常数。g_c对应于重叠振幅、连续谱阈值以及Borel变量在其工作区的中间值：$\omega_c=3.4$ GeV，$T=2.75$ GeV。

$$
\begin{aligned}
g^{p1}_{M_2H_1\rho} &= \frac{\sqrt{6}}{4}g^{p1}_{M_1H_0\rho} = -\frac{\sqrt{6}}{2}g^{p1}_{M_1H_1\rho}\,,\\
g^{p2}_{M_2H_1\rho} &= \frac{\sqrt{6}}{6}g^{p2}_{M_1H_1\rho} = -\frac{1}{2}g^{p2}_{M_2H_0\rho}\,,\\
g^{f2}_{M_2H_1\rho} &= \frac{\sqrt{6}}{6}g^{f2}_{M_1H_1\rho} = -\frac{1}{2}g^{f2}_{M_2H_0\rho}\,,\\
g^{s1}_{M_2S_1\rho} &= -\frac{\sqrt{6}}{4}g^{s1}_{M_1S_0\rho} = -\frac{\sqrt{6}}{2}g^{s1}_{M_1S_1\rho}\,,\\
g^{d1}_{M_2S_1\rho} &= -\frac{\sqrt{6}}{4}g^{d1}_{M_1S_0\rho} = \frac{\sqrt{6}}{2}g^{d1}_{M_1S_1\rho}\,,\\
g^{d2}_{M_2S_1\rho} &= -\frac{\sqrt{6}}{6}g^{d2}_{M_1S_1\rho} = -\frac{1}{2}g^{d2}_{M_2S_0\rho}\,.
\end{aligned}
\tag{9}
$$

π振幅的定义：

$$\begin{aligned}
\mathcal{M}(M_1 \to H_0 + \pi) &= I(\eta \cdot q_t) g^{p1}_{M_1 H_0 \pi} \,, \\
\mathcal{M}(M_1 \to H_1 + \pi) &= Ii\varepsilon^{\eta\epsilon^* qv} g^{p1}_{M_1 H_1 \pi} \,, \\
\mathcal{M}(M_1 \to S_1 + \pi) &= I\left[(\eta \cdot q_t)(\epsilon^* \cdot q_t) - \frac{1}{3}(\eta \cdot \epsilon^*_t) q_t^2\right] g^{d2}_{M_1 S_1 \pi} \,, \\
\mathcal{M}(M_1 \to T_1 + \pi) &= I(\eta \cdot \epsilon^*_t) g^{s0}_{M_1 T_1 \pi} + I\left[(\eta \cdot q_t)(\epsilon^* \cdot q_t) - \frac{q_t^2}{3}(\eta \cdot \epsilon^*_t)\right] g^{d2}_{M_1 T_1 \pi} \,, \\
\mathcal{M}(M_1 \to T_2 + \pi) &= 2Ii\epsilon^*_{\beta_1\beta_2} \varepsilon^{\beta_1 \eta q v} q_t^{\beta_2} g^{d2}_{M_1 T_2 \pi} \,, \\
\mathcal{M}(M_2 \to S_0 + \pi) &= I\eta_{\alpha_1\alpha_2}\left[q_t^{\alpha_1} q_t^{\alpha_2} - \frac{1}{3} g_t^{\alpha_1\alpha_2} q_t^2\right] g^{d2}_{M_2 S_0 \pi} \,, \\
\mathcal{M}(M_2 \to S_1 + \pi) &= Ii\eta_{\alpha_1\alpha_2} \epsilon^*_\beta \varepsilon^{\beta\alpha_1 qv} q_t^{\alpha_2} g^{d2}_{M_2 S_1 \pi} \,, \\
\mathcal{M}(M_2 \to T_1 + \pi) &= 2Ii\eta_{\alpha_1\alpha_2} \varepsilon^{\alpha_1 \epsilon^* qv} q_t^{\alpha_2} g^{d2}_{M_2 T_1 \pi} \,, \\
\mathcal{M}(M_2 \to T_2 + \pi) &= 2I\eta_{\alpha_1\alpha_2}\epsilon^*_{\beta_1\beta_2}\left[g_t^{\alpha_1\beta_1} g_t^{\alpha_2\beta_2} - \frac{1}{3} g_t^{\alpha_1\alpha_2} g_t^{\beta_1\beta_2}\right] g^{s0}_{M_2 T_2 \pi} \\
&\quad + I\eta_{\alpha_1\alpha_2}\epsilon^*_{\beta_1\beta_2}\left\{q_t^{\alpha_1} q_t^{\alpha_2} g_t^{\beta_1\beta_2} + q_t^{\beta_1} q_t^{\beta_2} g_t^{\alpha_1\alpha_2}\right. \\
&\quad \left. -3 q_t^{\alpha_1} q_t^{\beta_1} g_t^{\alpha_2\beta_2} - q_t^2\left[\frac{2}{3} g_t^{\alpha_1\alpha_2} g_t^{\beta_1\beta_2} - g_t^{\alpha_1\beta_1} g_t^{\alpha_2\beta_2}\right]\right\} g^{d2}_{M_2 T_2 \pi} \,,
\end{aligned}$$

$$
\begin{aligned}
\mathcal{M}(M_1 \to M_1 + \pi) &= Ii\varepsilon^{\eta\epsilon^* qv} g^{p1}_{M_1M_1\pi}\,, \\
\mathcal{M}(M_2 \to M_1 + \pi) &= 2I\eta_{\alpha_1\alpha_2}\left[\epsilon_t^{*\alpha_1} q_t^{\alpha_2} - \frac{1}{3} g_t^{\alpha_1\alpha_2}(\epsilon^* \cdot q_t)\right] g^{p1}_{M_2M_1\pi} \\
&\quad + I\eta_{\alpha_1\alpha_2}\left\{ q_t^{\alpha_1} q_t^{\alpha_2}(\epsilon^* \cdot q_t) - \frac{q_t^2}{5}\left[2\epsilon_t^{*\alpha_1} q_t^{\alpha_2} + (\epsilon^* \cdot q_t) g_t^{\alpha_1\alpha_2}\right]\right\} g^{f3}_{M_2M_1\pi}\,, \\
\mathcal{M}(M_2 \to M_2 + \pi) &= 4Ii\eta_{\alpha_1\alpha_2}\eta^*_{\beta_1\beta_2}\varepsilon^{\alpha_1\beta_1 qv} g_t^{\alpha_2\beta_2} g^{p1}_{M_2M_2\pi} \\
&\quad + 4Ii\eta_{\alpha_1\alpha_2}\eta^*_{\beta_1\beta_2}\varepsilon^{\alpha_1\beta_1 qv}\left[q_t^{\alpha_2} q_t^{\beta_2} - \frac{q_t^2}{5} g_t^{\alpha_2\beta_2}\right] g^{f3}_{M_2M_2\pi}\,. \qquad (10)
\end{aligned}
$$

ρ振幅的定义:

$$
\begin{aligned}
\mathcal{M}(M_1 \to H_0 + \rho) &= I\epsilon^{\eta e^* q v} g^{p1}_{M_1 H_0 \rho}\,, \\
\mathcal{M}(M_1 \to H_1 + \rho) &= Ii\left[(\eta\cdot e_t^*)(\epsilon^*\cdot q_t) - (\eta\cdot q_t)(\epsilon^*\cdot e_t^*)\right] g^{p1}_{M_1 H_1 \rho} \\
&\quad + Ii\left[(\eta\cdot e_t^*)(\epsilon^*\cdot q_t) + (\eta\cdot q_t)(\epsilon^*\cdot e_t^*) - \frac{2}{3}(\eta\cdot \epsilon_t^*)(e^*\cdot q_t)\right] g^{p2}_{M_1 H_1 \rho} \\
&\quad + Ii\Big\{(\eta\cdot q_t)(\epsilon^*\cdot q_t)(e^*\cdot q_t) - \frac{q_t^2}{5}\left[(\eta\cdot \epsilon_t^*)(e^*\cdot q_t) + (\eta\cdot e_t^*)(\epsilon^*\cdot q_t) + (\eta\cdot q_t)(\epsilon^*\cdot e_t^*\right. \\
\mathcal{M}(M_1 \to S_0 + \rho) &= Ii(\eta\cdot e_t^*) g^{s1}_{M_1 S_0 \rho} + Ii\left[(\eta\cdot q_t)(e^*\cdot q_t) - \frac{1}{3}(\eta\cdot e_t^*) q_t^2\right] g^{d1}_{M_1 S_0 \rho}\,, \\
\mathcal{M}(M_1 \to S_1 + \rho) &= I\epsilon^{\eta \epsilon^* e^* v} g^{s1}_{M_1 S_1 \rho} \\
&\quad + I\left[\epsilon^{\eta \epsilon^* q v}(e^*\cdot q_t) - \frac{1}{3}\epsilon^{\eta \epsilon^* e^* v} q_t^2\right] g^{d1}_{M_1 S_1 \rho} \\
&\quad + I\left[\epsilon^{\eta e^* q v}(\epsilon^*\cdot q_t) + \epsilon^{\epsilon^* e^* q v}(\eta\cdot q_t)\right] g^{d2}_{M_1 S_1 \rho}\,,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}(M_2 \to H_0 + \rho) &= 2Ii\eta_{\alpha_1\alpha_2}\left[e_t^{*\alpha_1}q_t^{\alpha_2} - \frac{1}{3}g_t^{\alpha_1\alpha_2}(e^* \cdot q_t)\right] g^{p2}_{M_2H_0\rho} \\
&+ Ii\eta_{\alpha_1\alpha_2}\left\{q_t^{\alpha_1}q_t^{\alpha_2}(e^* \cdot q_t) - \frac{q_t^2}{5}\left[g_t^{\alpha_1\alpha_2}(e^* \cdot q_t) + 2e_t^{*\alpha_1}q_t^{\alpha_2}\right]\right\} g^{f2}_{M_2H_0\rho}, \\
\mathcal{M}(M_2 \to H_1 + \rho) &= 2I\eta_{\alpha_1\alpha_2}\left[-\varepsilon^{\alpha_1 e^* qv}\epsilon_t^{*\alpha_2} + \frac{1}{3}g_t^{\alpha_1\alpha_2}\varepsilon^{\epsilon^* e^* qv}\right] g^{p1}_{M_2H_1\rho} \\
&+ 2I\eta_{\alpha_1\alpha_2}\left[\varepsilon^{\alpha_1\epsilon^* e^* v}q_t^{\alpha_2} + \varepsilon^{\alpha_1\epsilon^* qv}e_t^{*\alpha_2}\right] g^{p2}_{M_2H_1\rho} \\
&+ 2I\eta_{\alpha_1\alpha_2}\left\{\varepsilon^{\alpha_1\epsilon^* qv}q_t^{\alpha_2}(e^* \cdot q_t) - \frac{q_t^2}{5}\left[\varepsilon^{\alpha_1\epsilon^* qv}e_t^{*\alpha_2} + \varepsilon^{\alpha_1\epsilon^* e^* v}q_t^{\alpha_2}\right]\right\} g^{f2}_{M_2H_1\rho}, \\
\mathcal{M}(M_2 \to S_0 + \rho) &= 2I\eta_{\alpha_1\alpha_2}\epsilon^{\alpha_1 e^* qv}q_t^{\alpha_2}g^{d2}_{M_2S_0\rho}, \\
\mathcal{M}(M_2 \to S_1 + \rho) &= 2Ii\eta_{\alpha_1\alpha_2}\left[\epsilon_t^{*\alpha_1}e_t^{*\alpha_2} - \frac{1}{3}g_t^{\alpha_1\alpha_2}(\epsilon^* \cdot e_t^*)\right] g^{s1}_{M_2S_1\rho} \\
&+ 2Ii\eta_{\alpha_1\alpha_2}\left\{\left[\epsilon_t^{*\alpha_1}q_t^{\alpha_2} - \frac{1}{3}g_t^{\alpha_1\alpha_2}(\epsilon^* \cdot q_t)\right](e^* \cdot q_t) - \frac{q_t^2}{3}\left[\epsilon_t^{*\alpha_1}e_t^{*\alpha_2} - \frac{1}{3}g_t^{\alpha_1\alpha_2}(\epsilon^* \cdot\right.\right. \\
&+ 2Ii\eta_{\alpha_1\alpha_2}\left\{2\left[e_t^{*\alpha_1}q_t^{\alpha_2}(\epsilon^* \cdot q_t) - q_t^{\alpha_1}q_t^{\alpha_2}(\epsilon^* \cdot e_t^*)\right]\right. \\
&+ \left.\left[\epsilon_t^{*\alpha_1}q_t^{\alpha_2} - g_t^{\alpha_1\alpha_2}(\epsilon^* \cdot q_t)\right](e^* \cdot q_t) - \left[\epsilon_t^{*\alpha_1}e_t^{*\alpha_2} - g_t^{\alpha_1\alpha_2}(\epsilon^* \cdot e_t^*)\right]q_t^2\right\} g^{d2}_{M_2S_1\rho}.
\end{aligned}
$$

电磁衰变振幅

电磁耦合常数的计算可类似处理。由于光子静质量为零，耦合常数定义为$m1$，$e2\cdots$，如：

$$\begin{aligned}
& \mathcal{M}(M_1 \to H_1 + \gamma) \\
= \; & ei\left[(\eta \cdot e_t^*)(\epsilon^* \cdot q_t) - (\eta \cdot q_t)(\epsilon^* \cdot e_t^*)\right] g^{m1}_{M_1 H_1 \gamma} \\
& + ei\left\{(\eta \cdot q_t)(\epsilon^* \cdot q_t)(e^* \cdot q_t) \right. \\
& \left. - \frac{q_t^2}{2}\left[(\eta \cdot e_t^*)(\epsilon^* \cdot q_t) + (\eta \cdot q_t)(\epsilon^* \cdot e_t^*)\right]\right\} g^{e2}_{M_1 H_1 \gamma} \,, \qquad (12)
\end{aligned}$$

其中e为质子电荷。

关联函数：

为得出$g^{m1}_{M_1H_1\gamma}$和$g^{e2}_{M_1H_1\gamma}$的求和规则，考虑关联函数：

$$\int d^4x e^{-ik\cdot x}\langle\gamma(q)|T\{J^{\beta}_{1,-,\frac{1}{2}}(0)J^{\dagger\alpha}_{1,-,\frac{3}{2}}(x)\}|0\rangle$$

$$= ei\left[e^{*\alpha}_t q^{\beta}_t - e^{*\beta}_t q^{\alpha}_t\right] G^{m1}_{M_1H_1\gamma}(\omega,\omega')$$

$$+ei\left\{q^{\alpha}_t q^{\beta}_t (e^*\cdot q_t) - \frac{q^2_t}{2}\left[e^{*\alpha}_t q^{\beta}_t + e^{*\beta}_t q^{\alpha}_t\right]\right\} G^{e2}_{M_1H_1\gamma}(\omega,\omega') . \quad (13)$$

在重夸克极限下，光子和轻夸克有两种耦合方式：

- QED相互作用。此时需要夸克在外电磁场下的传播子：

$$\begin{aligned}&\langle 0|T\{q^a(x)\bar{q}^b(0)\}|0\rangle_{F_{\mu\nu}}\\ =\ &\frac{\delta^{ab}e_q e}{16\pi^2x^2}\int_0^1 du\{2(1-2u)x_\mu\gamma_\nu+i\varepsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\rho x^\sigma\}F^{\mu\nu}(ux)\,,\end{aligned} \tag{14}$$

  此处利用Fock-Schwinger规范$x^\mu A_\mu(x)=0$将电磁势A用规范不变的场强$F_{\mu\nu}$表示出来。

- 通过光子的光锥分布振幅表达的非微扰相互作用。

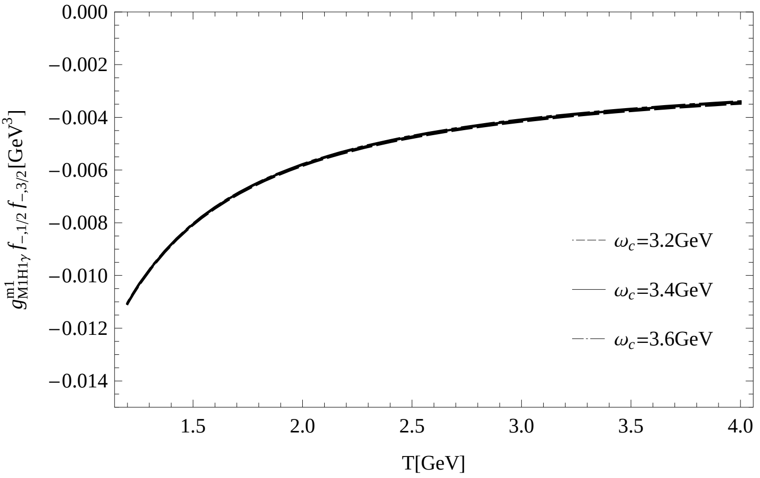

Figure: The sum rule for $g^{m1}_{M_1 H_1 \gamma} f_{-,1/2} f_{-,3/2}$ with $\omega_c = 3.2, 3.4, 3.6$ GeV and the working interval $2.5 < T < 3.5$ GeV.

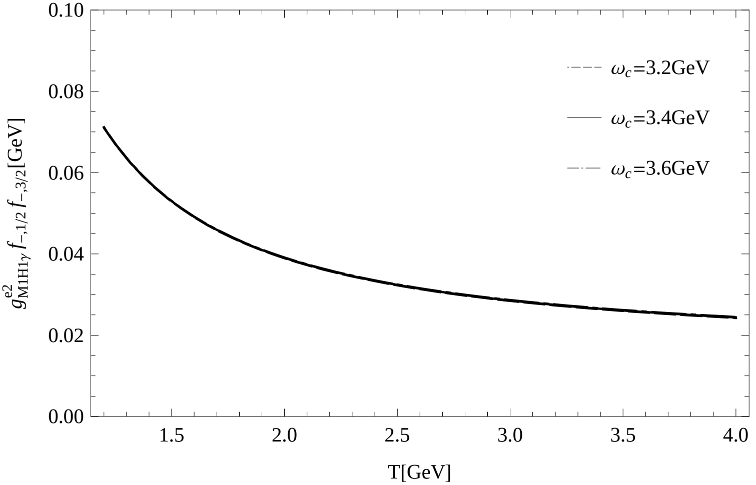

Figure: The sum rule for $g^{e2}_{M_1H_1\gamma} f_{-,1/2} f_{-,3/2}$ with $\omega_c = 3.2, 3.4, 3.6$ GeV and the working interval $2.0 < T < 2.4$ GeV.

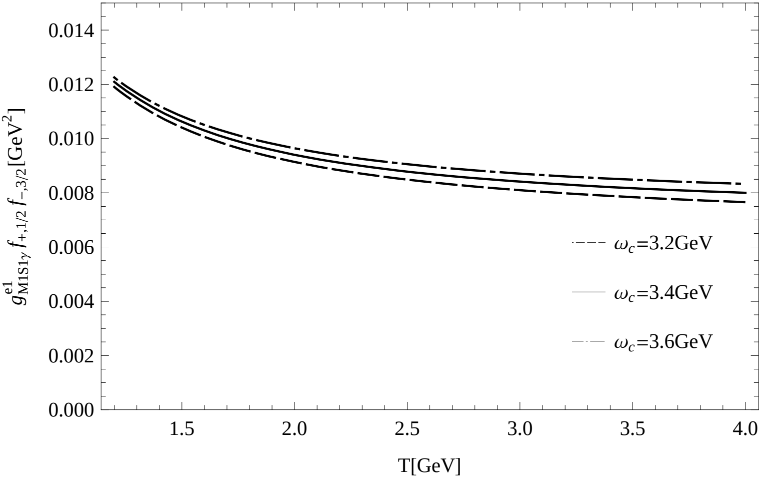

Figure: The sum rule for $g^{e1}_{M_1 S_1 \gamma} f_{+,1/2} f_{-,3/2}$ with $\omega_c = 3.2, 3.4, 3.6$ GeV and the working interval $2.5 < T < 3.5$ GeV.

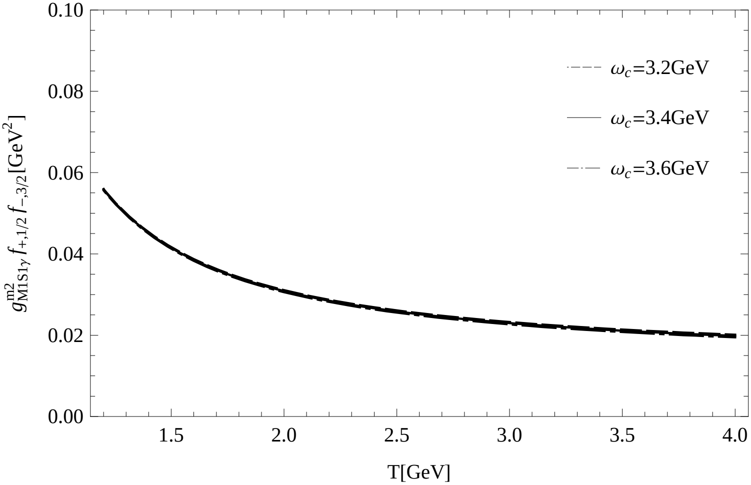

Figure: The sum rule for $g^{m2}_{M_1 S_1 \gamma} f_{+,1/2} f_{-,3/2}$ with $\omega_c = 3.2, 3.4, 3.6$ GeV and the working interval $2.5 < T < 3.5$ GeV.

输入参数：γ光锥波函数中的参数和凝聚量($\mu = 1$GeV)

$f_{3\gamma}$	ω_γ^V	ω_γ^A	κ	ζ_1	ζ_2
$-0.0039\mathrm{GeV}^2$	3.8 ± 1.8	-2.1 ± 1.0	0.2	0.4	0.3
φ_2	κ^+	ζ_1^+	ζ_2^+	$\langle \bar{q}q \rangle$	χ
0	0	0	0	$(-0.245\,\mathrm{GeV})^3$	$(3.15 \pm 0.3)\,\mathrm{GeV}^{-2}$

P. Ball and G. W. Jones, JHEP **0703**, 069 (2007); P. Ball, V. M. Braun, and A. Lenz, JHEP **0708**, 090 (2007); P. Ball, V. M. Braun, and N. Kivel, Nucl. Phys. **B649**, 263 (2003); I. I. Balitsky, V. M. Braun, and A. V. Kolesnichenko, Sov. J. Nucl. Phys. **48**, 348 (1988), [Yad. Fiz. **48**, 547 (1988)]; Nucl. Phys. **B312**, 509 (1989).

电磁耦合常数的数值结果：

	$g^{m1}_{M_1H_1\gamma}$ [GeV^{-1}]	$g^{e2}_{M_1H_1\gamma}$ [GeV^{-3}]	$g^{e1}_{M_1S_1\gamma}$ [GeV^{-2}]	$g^{m2}_{M_1S_1\gamma}$ [GeV^{-2}]
g_c	-0.05	0.37	0.06	0.16
g	$-0.07 \sim -0.03$	$0.26 \sim 0.53$	$0.04 \sim 0.10$	$0.10 \sim 0.28$

Table: 电磁耦合常数。g_c对应于重叠振幅、连续谱阈值以及Borel变量在其工作区的中间值：$\omega_c = 3.4$ GeV，$T = 2.75$ GeV。

$$
\begin{aligned}
g^{m1}_{M_1H_1\gamma} &= -\frac{1}{2}g^{m1}_{M_1H_0\gamma} = -\frac{\sqrt{6}}{3}g^{m1}_{M_2H_1\gamma}\,, \\
g^{e2}_{M_1H_1\gamma} &= -\frac{\sqrt{6}}{2}g^{e2}_{M_2H_0\gamma} = \sqrt{6}g^{e2}_{M_2H_1\gamma}\,, \\
g^{e1}_{M_1S_1\gamma} &= -\frac{1}{2}g^{e1}_{M_1S_0\gamma} = \frac{\sqrt{6}}{3}g^{e1}_{M_2S_1\gamma}\,, \\
g^{m2}_{M_1S_1\gamma} &= \frac{\sqrt{6}}{2}g^{m2}_{M_2S_0\gamma} = -\sqrt{6}g^{m2}_{M_2S_1\gamma}\,. \qquad (15)
\end{aligned}
$$

γ振幅的定义:

$$\begin{aligned}
\mathcal{M}(M_1 \to H_0 + \gamma) &= e\varepsilon^{\eta e^* qv} g^{m1}_{M_1 H_0 \gamma} \,, \\
\mathcal{M}(M_1 \to S_0 + \gamma) &= ei\left[(\eta \cdot q_t)(e^* \cdot q_t) - (\eta \cdot e^*_t) q_t^2\right] g^{e1}_{M_1 S_0 \gamma} \,, \\
\mathcal{M}(M_1 \to S_1 + \gamma) &= e\left[\varepsilon^{\eta \epsilon^* e^* v} q_t^2 - \varepsilon^{\eta \epsilon^* qv}(e^* \cdot q_t)\right] g^{e1}_{M_1 S_1 \gamma} \\
&\quad + e\left[2\varepsilon^{\eta e^* qv}(\epsilon^* \cdot q_t) + \varepsilon^{\eta \epsilon^* e^* v} q_t^2 - \varepsilon^{\eta \epsilon^* qv}(e^* \cdot q_t)\right] g^{m2}_{M_1 S_1 \gamma} \,, \\
\mathcal{M}(M_2 \to H_0 + \gamma) &= ei\eta_{\alpha_1 \alpha_2}\left[q_t^{\alpha_1} q_t^{\alpha_2}(e^* \cdot q_t) - e^{*\alpha_1}_t q_t^{\alpha_2} q_t^2\right] g^{e2}_{M_2 H_0 \gamma} \,, \\
\mathcal{M}(M_2 \to H_1 + \gamma) &= 2e\eta_{\alpha_1 \alpha_2}\left[\varepsilon^{\alpha_1 \epsilon^* qv} e^{*\alpha_2}_t - \varepsilon^{\alpha_1 \epsilon^* e^* v} q_t^{\alpha_2} + \frac{2}{3} g_t^{\alpha_1 \alpha_2} \varepsilon^{\epsilon^* e^* qv}\right] g^{m1}_{M_2 H_1 \gamma} \\
&\quad + e\eta_{\alpha_1 \alpha_2}\left\{q_t^2\left[\varepsilon^{\alpha_1 \epsilon^* qv} e^{*\alpha_2}_t + \varepsilon^{\alpha_1 \epsilon^* e^* v} q_t^{\alpha_2}\right] - 2\varepsilon^{\alpha_1 \epsilon^* qv} q_t^{\alpha_2}(e^* \cdot q_t)\right\} g^{e2}_{M_2 H_1 \gamma} \,, \\
\mathcal{M}(M_2 \to S_0 + \gamma) &= 2e\eta_{\alpha_1 \alpha_2} \varepsilon^{\alpha_1 e^* qv} q_t^{\alpha_2} g^{m2}_{M_2 S_0 \gamma} \,,
\end{aligned}$$

$$\begin{aligned}
&\mathcal{M}(M_2 \to S_1 + \gamma) \\
&= 2ei\eta_{\alpha_1 \alpha_2}\left\{\left[\epsilon^{*\alpha_1}_t q_t^{\alpha_2} - \frac{1}{3} g_t^{\alpha_1 \alpha_2}(\epsilon^* \cdot q_t)\right](e^* \cdot q_t) - \left[\epsilon^{*\alpha_1}_t e^{*\alpha_2}_t - \frac{1}{3} g_t^{\alpha_1 \alpha_2}(\epsilon^* \cdot e^*_t)\right] q_t^2\right\} g^{e1}_{M_2 S_1 \gamma} \\
&\quad + 2ei\eta_{\alpha_1 \alpha_2}\left\{2\left[e^{*\alpha_1}_t q_t^{\alpha_2}(\epsilon^* \cdot q_t) - q_t^{\alpha_1} q_t^{\alpha_2}(\epsilon^* \cdot e^*_t)\right] + \left[\epsilon^{*\alpha_1}_t q_t^{\alpha_2} - g_t^{\alpha_1 \alpha_2}(\epsilon^* \cdot q_t)\right](e^* \cdot q_t)\right. \\
&\quad \left. - \left[\epsilon^{*\alpha_1}_t e^{*\alpha_2}_t - g_t^{\alpha_1 \alpha_2}(\epsilon^* \cdot e^*_t)\right] q_t^2\right\} g^{m2}_{M_2 S_1 \gamma} \,. \qquad (16)
\end{aligned}$$

质量参数：

J^P	0^-	1^-	0^+	1^+	1^+	2^+	1^-	2^-
m_D [GeV]	1.87	2.01	2.40	2.43	2.42	2.46	2.82	2.83
m_B [GeV]	5.28	5.33	5.70	5.73	5.72	5.75	6.10	6.11
m_{D_s} [GeV]	1.97	2.11						
m_{B_s} [GeV]	5.37	5.41						

Table: 衰变宽度计算中用到的重味介子质量. m_D, m_{D^*}, $m_{D_0^*}$, $m_{D_1'}$, m_{D_1}, m_{D_2}, m_B, m_{B^*}, $m_{B_1'}$, m_{B_1}, m_{B_2}, m_{D_s}和m_{B_s}取自PDG 2008，其余来自夸克模型估计(S. Godfrey and N. Isgur, PRD32, 189 (1985).)

单π衰变(D介子)：

	$D\pi^+$	$D^*\pi^+$	$D_0^*\pi^+$	$D_1'\pi^+$	$D_1\pi^+$	$D_2\pi^+$
$D_1^{*\prime}\rightarrow$	3.7	1.3		0.02	2.1	0.04
	1.8 — 7.9	0.6 — 2.9		0.01 — 0.04	0.3 — 8.9	0.02 — 0.1
$D_2^*\rightarrow$		4.4	0.02	0.02	0.04	7.8
		2.1 — 9.3	0.01 — 0.05	0.01 — 0.06	0.02 — 1.0	1.1 — 33.5

	$D\eta$	$D^*\eta$	D_sK^+	$D_s^*K^+$	$\Gamma_{D\rightarrow D+P}$
$D_1^{*\prime}\rightarrow$	0.3	0.1	1.6	0.4	13.1
	0.2 — 0.7	0.1 — 0.2	0.8 — 3.4	0.2 — 0.8	5.4 — 34.9
$D_2^*\rightarrow$		0.3		1.2	19.9
		0.1 — 0.6		0.6 — 2.6	5.6 — 69.1

Table: D介子单π衰变宽度(单位：MeV)。

单π衰变(B介子):

	$B\pi^+$	$B^*\pi^+$	$B_0^*\pi^+$	$B_1'\pi^+$	$B_1\pi^+$	$B_2\pi^+$	
$B_1^{*\prime} \to$	4.4	1.9		0.02	2.3	0.05	
	2.1 – 9.3	0.9 – 3.9		0.01 – 0.04	0.4 – 9.6	0.02 – 0.11	
$B_2^* \to$		5.8	0.02	0.02	0.05	8.5	
		2.8 – 12.4	0.01 – 0.06	0.01 – 0.06	0.02 – 0.11	1.2 – 36.3	
	$B\eta$	$B^*\eta$	B_sK^+	$B_s^*K^+$			$\Gamma_{B\to B+P}$
$B_1^{*\prime} \to$	0.3	0.1	1.3	0.5			15.2
	0.1 – 0.6	0.1 – 0.2	0.6 – 2.8	0.2 – 1.0			6.1 – 39.0
$B_2^* \to$		0.3		1.6			23.5
		0.2 – 0.7		0.7 – 3.3			7.0 – 77.4

Table: B介子单π衰变宽度(单位：MeV)。

双π衰变(D介子):

	$D\pi^+\pi^0$	$D^*\pi^+\pi^0$	$D_0^*\pi^+\pi^0$	$D_1'\pi^+\pi^0$
$D_1^{*'} \to$	44.5	642.3	3.0	1.2
	7.1 – 178.2	241.3 – 1529.8	1.0 – 8.5	0.4 – 3.4
$D_2^* \to$	1155.0	448.3	0.06	3.5
	440.6 – 2728.3	165.1 – 1092.8	0.03 – 0.1	1.1 – 10.0

Table: D介子双π衰变宽度(单位：keV)。

双π衰变(B介子):

	$B\pi^+\pi^0$	$B^*\pi^+\pi^0$	$B_0^*\pi^+\pi^0$	$B_1'\pi^+\pi^0$
$B_1^{*\prime} \to$	19.1	579.7	2.7	1.1
	3.1 – 76.5	217.3 – 1382.1	0.8 – 7.6	0.3 – 3.0
$B_2^* \to$	521.7	414.2	0.05	3.1
	198.1 – 1234.1	152.5 – 1008.4	0.02 – 0.12	1.0 – 8.7

Table: B介子双π衰变宽度(单位：keV)。

辐射衰变(D介子):

	$D\gamma$	$D^*\gamma$	$D_0^*\gamma$	$D_1'\gamma$
$D_1^{*'} \to$	8.0	12.2	0.3	0.8
	$2.9 - 15.6$	$5.6 - 24.7$	$0.1 - 0.7$	$0.3 - 2.3$
$D_2^* \to$	9.0	15.7	0.4	0.8
	$4.4 - 18.5$	$6.5 - 31.3$	$0.2 - 1.3$	$0.3 - 2.3$

Table: D介子辐射衰变宽度(单位：keV)。

辐射衰变(B介子):

	$B\gamma$	$B^*\gamma$	$B_0^*\gamma$	$B_1'\gamma$
$B_1^{*\prime} \to$	9.4	18.8	0.3	0.8
	3.4 – 18.5	8.8 – 38.3	0.1 – 0.8	0.3 – 2.4
$B_2^* \to$	9.5	22.2	0.5	0.8
	4.7 – 19.6	9.3 – 44.5	0.2 – 1.4	0.3 – 2.4

Table: B介子辐射衰变宽度(单位：keV)。

结论:

- 在重夸克极限下用QCD光锥求和规则系统计算了重味介子和轻介子以及光子的耦合常数，得到的大部分求和规则是稳定的。
- 由此得到的$D(B)$介子的衰变宽度相当小。
- 对于B介子，可以预期这是好的近似；而对于D介子，也许需要考虑其有限质量带来的修正。

谢谢！