NNLO corrections to rare and hadronic B-meson decays

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Outline

1 Introduction

2 Theoretical framework for hadronic B decays

3 Status of NNLO calculation for $T_i^{I,II}$

4 Phenomenological applications

5 Matching heavy-to-light currents to NNLO in SCET

6 Conclusion and outlook

Overview of rare and hadronic B-meson decays

- B-meson weak decays play a very important role in:
  - testing the Standard Model;
  - probing the origin of CP violation;
  - searching for indirect signals of New Physics.
- Exp. side: more data and more precise due to:
  - BaBar at SLAC and Belle at KEK;
  - Tevatron at Fermilab and LHC-b at CERN;
  - higher luminosity Super-B factory.
- Theo. side: various theoretical frameworks proposed:
  - methods based on flavour symmetries of QCD;
  - methods based on factorization theorems of QCD dynamics: PQCD, QCDF and SCET;
  - non-perturbative methods, Lattice QCD or QCDSR;

B physics at the NNLO frontier: current status

- NNLO program for $\mathcal{H}_{eff} = \sum_i C_i Q_i$ now complete:
  - ▷ 2-loop/3-loop matching corrections [Bobeth, Misiak, Urban 99; Misiak, Steinhauser 04]
  - ▷ 3-loop/4-loop anomalous dimensions [Gorbahn, Haisch 04; Gorbahn, Haisch, Misiak 05; Czakon, Haisch, Misiak 06]

$\Rightarrow$ *need to calculate the hadronic matrix elements to the same level of precision!*

- Some NNLO analysis of B-meson decays:
  - ▷ $B \to X_s \gamma$ [M. Misiak *et al.* 06; Becher, Neubert 06]
  - ▷ $B \to X_u \ell \nu$ [Greub, Neubert, Pecjak 09]
  - ▷ $B \to M_1 M_2$ [Beneke, Jaeger 05,06; Bell 07,09; Jain, Rothstein, Stewart 07; Beneke, Huber, X. Q. Li 09 X. Q. Li, Y. D. Yang 05,06]

The effective weak Hamiltonian

- Effective weak Hamiltonian: [BBL basis *Buras,Buchalla,Lautenbacher'96*; CMM basis *Chetyrkin, Misiak, Münz'98*]

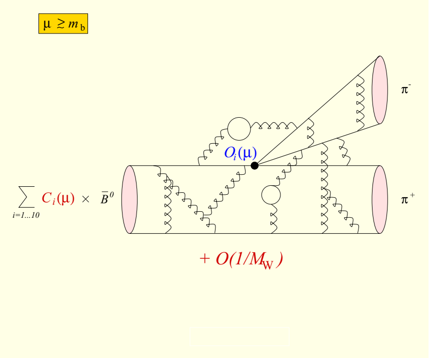
$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{k=3}^{6} C_k Q_k + C_7 Q_7 + C_8 Q_8 \right] + \text{h.c.}$$

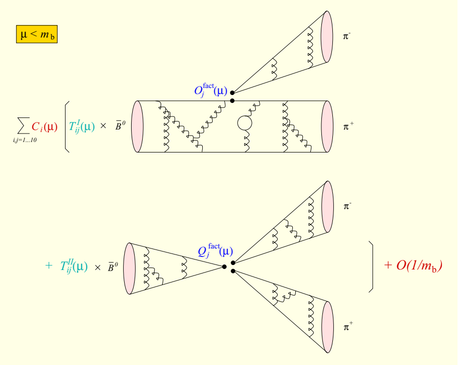
- C_i: include physics from m_W down to m_b scale; perturbatively calculable; have been calculated to NNLO [*Gorbahn and Haisch 04*].

- adopting CMM basis: convenient for multi-loop calculation; can safely use NDR scheme with anti-commuting γ_5.

- in NDR scheme, needs include evanescent operators. [*Gorbahn and Haisch 04*].
  -vanishing in 4 dim., important in the intermediate step,
  -needed to complete the operator basis under renormalization

QCD factorization approach





- BBNS factorization formula: [*Beneke,Buchalla,Neubert,Sachrajda'99-'04*]

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq F_+^{BM_1}(0)\, f_{M_2} \int du\; T_i^I(u)\, \phi_{M_2}(u)$$

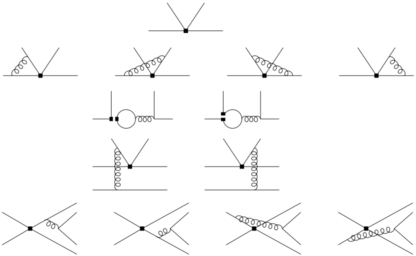
$$+ \hat{f}_B\, f_{M_1}\, f_{M_2} \int d\omega dv du\; T_i^{II}(\omega, v, u)\, \phi_B(\omega)\, \phi_{M_1}(v)\, \phi_{M_2}(u)$$

Perturbative calculation of hard scattering kernels $T^{I,II}$

- $T^{I,II}$: perturbative calculable order by order in α_s;

$$T^{I} = \mathcal{O}(1), \text{ while } T^{II} = \mathcal{O}(\alpha_s).$$

- relevant Feynman diagram at NLO in α_s:



- At NLO, all the relevant 130 two-body charmless decay modes have been analyzed:

BBNS; BN'03; Du; Cheng, for $B \to PP, PV$;

BRY'07; Kagan; Li and Yang; Cheng and Yang, for $B \to VV$;

......

Factorization formulae for $B \to M_1 M_2$ in SCET, I

- Soft-collinear effective thoery:
  - an EFT describing energetic ($E \gg \Lambda_{QCD}$) hadrons/jets;
  - suitable for studying factorization, resummation and power corrections;
  - successfully applied to $B \to X_s \gamma$, $B \to D\pi$, $B \to X_u \ell \nu$, $B \to X_s \ell^+ \ell^-$;
- two different formulations:

  1, SCET in momentum space, Hybrid expanded:

  C. W. Bauer, S. Fleming, D. Pirjol and I. W. Stewart, 00

  C. W. Bauer, D. Pirjol and I. W. Stewart, 01

  2, SCET in position space, multi-pole expanded:

  M. Beneke, A. P. Chapovsky, M. Diehl and Th. Feldmann, 02

  M. Beneke and Th. Feldmann, 02

  R. J. Hill and M. Neubert, 02

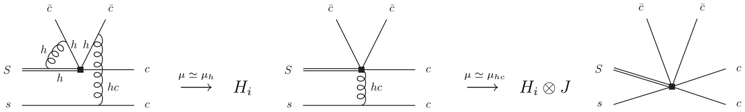
- Both are equivalent at all order, but do not coincide order by order in λ.

Factorization formulae for $B \to M_1 M_2$ in SCET, II

- $T^{I,II}$: more transparent in SCET; just operator matching calculations;
  matching coefficients of some SCET (non) local operators;

- Matching procedure for T^{II}: complicated due to μ_{hc}

$$\text{QCD} \;\to\; \text{SCET}_\text{I}(hc, c, s) \;\to\; \text{SCET}_\text{II}(c, s)$$

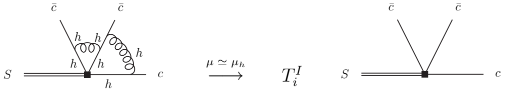


- Final result for T_i^{II}: $T_i^{II}(\omega, v, u) = \int dz\, J(\omega, v, z)\, H_i(z, u)$
  - $H_i = \mathcal{O}(1)$: hard coefficient, QCD $\to$ SCET$_\text{I}$ at $\mu_h \sim m_b$;
  - $J = \mathcal{O}(\alpha_s)$: jet function, SCET$_\text{I}$ $\to$ SCET$_\text{II}$ at $\mu_{hc} \sim \sqrt{m_b \Lambda_\text{QCD}}$;
  - resummation of $\log \mu_h/\mu_{hc}$ using RGEs in SCET; [*Beneke,Yang 05*]

Factorization formula for $B \to M_1 M_2$ in SCET, III

- Matching procedure for T^I: conceptually simpler, no intermediate scale μ_{hc};

$$\text{QCD} \;\to\; \text{SCET}_\text{I}(hc, c, s)$$



- Final result for the hard kernel T_i^I:

$$T_i^I(u) = T_i^{I(0)}(u) + \alpha_s\, T_i^{I(1)}(u) + \alpha_s^2\, T_i^{I(2)}(u)$$

- $T_i^{I(0)}$: naive factorization
- $T_i^{I(1)}$: NLO vertex correction
- $T_i^{I(2)}$: NNLO vertex correction, true 2-loop calculation.

Motivation for NNLO calculation

- Final results for $T_i^{I,II}$:

$$T_i^I(u) = T_i^{I(0)}(u) + \alpha_s\, T_i^{I(1)}(u) + \alpha_s^2\, T_i^{I(2)}(u)$$

$$T_i^{II}(\omega, v, u) = \int dz\, J(\omega, v, z)\, H_i(z, u)$$

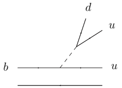
- Phenomenologically very relevant:
  - only the first corrections to strong phases, quite relevant to direct $C\not{P}$;
  - needed to reduce the large (N)LO scale uncertainties, generally expected;

- Current data driven: C/T or α_2 seems to be too small, large cancelation in LO + NLO, particularly sensitive to NNLO, $\Longrightarrow$enhancement from NNLO?.

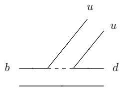
- Conceptual and systematic aspects:
  - verification of factorization at NNLO, still hold at NNLO?
  - spectator scatterings involve μ_{hc}, PT well-behaved?

Status of NNLO calculation

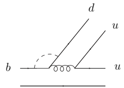
- Define the topological amplitudes:



colour-allowed tree α_1



colour-suppressed tree α_2

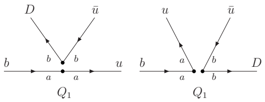


QCD penguins α_4

- Available NNLO corrections:
  - J: matching+resummation,
    [*Beneke,Yang 05; Becher,Hill,Lee,Neubert 04; Kirilin 05*]
  - H_i: for tree amplitudes, [*Beneke,Jaeger 05; Kivel 06; Pilipp 07*]
    for penguin amplitudes,
    [*Beneke,Jaeger 06; Li and Yang 06; Jain,Rothstein,Stewart 07*]
  - T^I: for tree amplitudes,
    [*Bell 06 (Im part), 09 (Re part); Beneke,Huber,Li, 09*]
    for penguin amplitudes, [*more complicated, to be done next!*]

NNLO vertex corrections to T^I

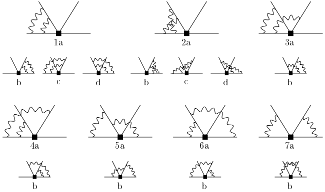
- due to the absence of the intermediate scale μ_{hc}, conceptually simpler;
- technically demanding, a genuine 2-loop calculation with four external legs.
- direct diagrammatical approach [*G.Bell 07, 09*]$\rightleftharpoons$ matching calculation within SCET [*M. Beneke, T. Huber, X.Q. Li 09*] $\Rightarrow$ *an important cross-check!*
- two different contraction of current-current operators:

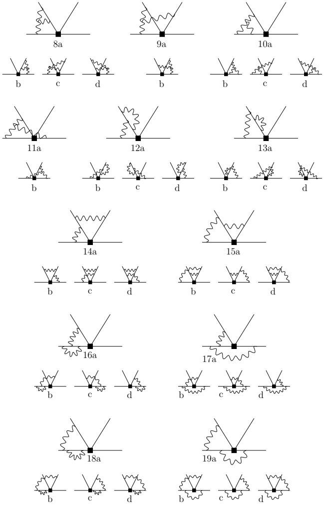


- two main tasks: calculating 2-loop QCD Feynman diagrams, performing complicated IR-subtraction.

Two-loop Feynman diagrams

- Two-loop non-factorizable diagrams:





- totally 62 “non-factorizable” diagrams;
- vacuum polarization insertions in gluon propagators;
- external self-energy insertions in quark propagators;
- one-loop counter-term insertions;

Multi-loop calculations in a nutshell

- Work in DR with $D = 4 - 2\epsilon$, to regulate both UV and IR div.; at 2-loop order, IR poles appear up to $1/\epsilon^4$.
- Basis strategy:
  - general tensor decomposition via Passarino-Veltman ansatz, $\Longrightarrow$thousands of scalar integrals, [*Passarino,Veltman'79*];
  - reduction them to Master Integral via Laporta algorithm based on Integration-by-part (IBP) identities$\Longrightarrow$totally 42 MIs, [*Tkachov'81; Chetyrkin,Tkachov'81*] [*Laporta'01; Anastasiou,Lazopoulos'04; Smirnov'08*];
  - calculation of the MIs, very challenging, all should be given analytically.
- Techniques used to calculate MIs:
  - standard Feynman parameterisation, only for simpler MIs;
  - method of differential equations, [*Kotikov'91; Remiddi'97*];
  - Mellin-Barnes techniques, [*Smirnov'99; Tausk'99*];

Master formula for hard scattering kernel in RI

$$
\begin{aligned}
T_i^{(1)} &= A_{i1}^{(1),nf} + Z_{ij}^{(1)} A_{j1}^{(0)} , \\
T_i^{(2)} &= A_{i1}^{(2),nf} + Z_{ij}^{(1)} A_{j1}^{(1)} + Z_{ij}^{(2)} A_{j1}^{(0)} + Z_{\alpha}^{(1)} A_{i1}^{(1),nf} + (-i)\, \delta_m^{(1)} A_{i1}'^{(1),nf} \\
&\quad + T_i^{(1)} \left[- C_{FF}^{(1)} - Y_{11} + Z_{ext}^{(1)} \right] - \sum_{b>1} H_{ib}^{(1)} Y_{b1}^{(1)} .
\end{aligned}
$$

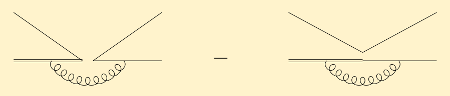
- $T_i^{(1)}$ and $T_i^{(2)}$: 1-loop and 2-loop hard scattering kernel;
- important check: both are free of poles in ϵ, factorization holds.
- needs higher-order ϵ terms in $T_i^{(1)}$ to compute $T_i^{(2)}$;
- Z_{ij}: renormalisation constants of QCD operators;
- Y_{ij}: renormalization constants of the SCET operators;
- C_{FF}: matching coefficient of QCD current to SCET current;
- needs also 1-loop factorizable diagrams;

Master formula for hard scattering kernel in WI

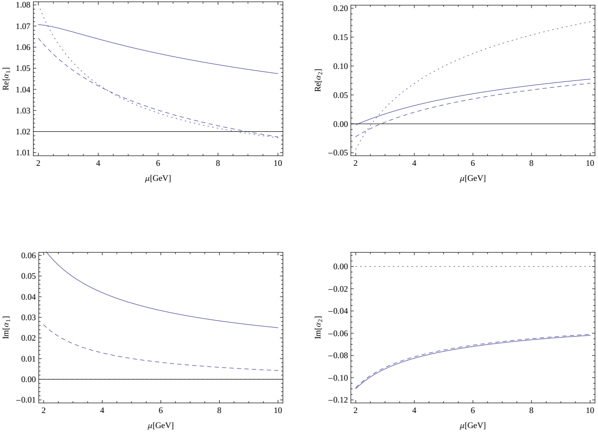
$$\widetilde{T}_i^{(1)} = \widetilde{A}_{i1}^{(1),nf} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)} + \underbrace{\widetilde{A}_{i1}^{(1),f} - A^{(1),f}\,\widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} - \underbrace{[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}]\,\widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)}$$

$$\begin{aligned}\widetilde{T}_i^{(2)} = {} & \widetilde{A}_{i1}^{(2),nf} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)}\,\widetilde{A}_{j1}^{(0)} + Z_{\alpha}^{(1)}\,\widetilde{A}_{i1}^{(1),nf} \\ & + (-i)\,\delta_m^{(1)}\,\widetilde{A}_{i1}'^{(1),nf} + Z_{ext}^{(1)}\,[\widetilde{A}_{i1}^{(1),nf} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)}] \\ & -\widetilde{T}_i^{(1)}\,[C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}] - \sum_{b>1}\widetilde{H}_{ib}^{(1)}\,\widetilde{Y}_{b1}^{(1)} \\ & +[\widetilde{A}_{i1}^{(2),f} - A^{(2),f}\,\widetilde{A}_{i1}^{(0)}] + (-i)\,\delta_m^{(1)}\,[\widetilde{A}_{i1}'^{(1),f} - A'^{(1),f}\,\widetilde{A}_{i1}^{(0)}] \\ & +(Z_{\alpha}^{(1)} + Z_{ext}^{(1)} + \xi_{45}^{(1)})[\widetilde{A}_{i1}^{(1),f} - A^{(1),f}\,\widetilde{A}_{i1}^{(0)}] \\ & -C_{FF}^{(1)}\,\widetilde{A}_{i1}^{(0)}[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}] - [\widetilde{Y}_{11}^{(2)} - Y_{11}^{(2)}]\,\widetilde{A}_{i1}^{(0)}\end{aligned}$$

- Fierz[$\tilde{O}_1$]=O_1 in $D = 4$ dim., so $\tilde{O}_1 - O_1$ is also evanescent;



Dependence of $\alpha_{1,2}$ on the hard scale μ_h, only vertex part!



- dotted line: LO result
- dashed line: NLO result
- solid line: NNLO result

- the real parts on the scale dependence substantially reduced!
- the imaginary parts less pronounced, since the NNLO term is really NLO!

Numerical result for α_1 and α_2

$$
\begin{aligned}
\alpha_1(\pi\pi) &= 1.009 + [0.023 + 0.010\, i]_{\rm NLO} + [0.026 + 0.028\, i]_{\rm NNLO} \\
&\quad - \left[\frac{r_{\rm sp}}{0.445}\right] \left\{ [0.014]_{\rm LOsp} + [0.034 + 0.027 i\,]_{\rm NLOsp} + [0.008]_{\rm tw3} \right\} \\
&= 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050})i
\end{aligned}
$$

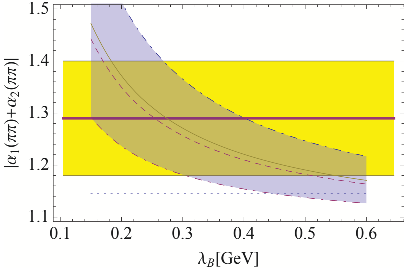
$$
\begin{aligned}
\alpha_2(\pi\pi) &= 0.220 - [0.179 + 0.077\, i]_{\rm NLO} - [0.031 + 0.050\, i]_{\rm NNLO} \\
&\quad + \left[\frac{r_{\rm sp}}{0.445}\right] \left\{ [0.114]_{\rm LOsp} + [0.049 + 0.051 i\,]_{\rm NLOsp} + [0.067]_{\rm tw3} \right\} \\
&= 0.240^{+0.217}_{-0.125} + (-0.077^{+0.115}_{-0.078})i
\end{aligned}
$$

- The NNLO corrections to the vertex term and spectator scattering are significant individually. But both tend to cancel each other. too bad!
- Largest uncertainty from $r_{\rm sp} = \frac{9 f_\pi \hat{f}_B}{m_b f_+^{B\pi}(0)\lambda_B}$, especially λ_B (B LCDA).
- Perturbation theory works at scales m_b and $\mu_{hc} = \sqrt{\Lambda_{\rm QCD} m_b}$.

Factorization test with sem-leptonic data

$$R_\pi \equiv \frac{\Gamma(B^- \to \pi^-\pi^0)}{d\Gamma(\bar{B}^0 \to \pi^+ l^- \bar{\nu})/dq^2\big|_{q^2=0}} = 3\pi^2 f_\pi^2 \, |V_{ud}|^2 \, |\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|^2$$

- From sem-leptonic data $[\,|\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|\,]_{\rm exp} = 1.29 \pm 0.11$
- Prediction with $\lambda_B = 0.35$ GeV: $|\alpha_1(\pi\pi) + \alpha_2(\pi\pi)| = 1.24^{+0.16}_{-0.10}$
- Good agreements observed, supporting QCD factorization.
- Main uncertainties: λ_B, α_2^π and power corrections.



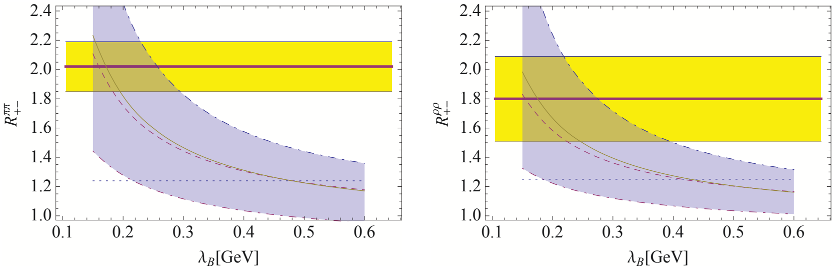
- interesting to extend to other final state; $R_\rho = 1.75^{+0.37}_{-0.24}$ $(2.08^{+0.50}_{-0.46})$;

Branching ratios for tree-dominated B decays

	Theory I	Theory II	Experiment
$B^- \to \pi^-\pi^0$	$5.43^{+0.06}_{-0.06}{}^{+1.45}_{-0.84}$	$5.82^{+0.07}_{-0.06}{}^{+1.42}_{-1.35}$	$5.59^{+0.41}_{-0.40}$
$\bar{B}_d^0 \to \pi^+\pi^-$	$7.37^{+0.86}_{-0.69}{}^{+1.22}_{-0.97}$	$5.70^{+0.70}_{-0.55}{}^{+1.16}_{-0.97}$	5.16 ± 0.22
$\bar{B}_d^0 \to \pi^0\pi^0$	$0.33^{+0.11}_{-0.08}{}^{+0.42}_{-0.17}$	$0.63^{+0.12}_{-0.10}{}^{+0.64}_{-0.42}$	1.55 ± 0.19
$B^- \to \pi^-\rho^0$	$8.68^{+0.42}_{-0.41}{}^{+2.71}_{-1.56}$	$9.84^{+0.41}_{-0.40}{}^{+2.54}_{-2.52}$	$8.3^{+1.2}_{-1.3}$
$B^- \to \pi^0\rho^-$	$12.38^{+0.90}_{-0.77}{}^{+2.18}_{-1.41}$	$12.13^{+0.85}_{-0.73}{}^{+2.23}_{-2.17}$	$10.9^{+1.4}_{-1.5}$
$\bar{B}^0 \to \pi^+\rho^-$	$17.80^{+0.62}_{-0.56}{}^{+1.76}_{-2.10}$	$13.76^{+0.49}_{-0.44}{}^{+1.77}_{-2.18}$	15.7 ± 1.8
$\bar{B}^0 \to \pi^-\rho^+$	$10.28^{+0.39}_{-0.39}{}^{+1.37}_{-1.42}$	$8.14^{+0.34}_{-0.33}{}^{+1.35}_{-1.49}$	7.3 ± 1.2
$\bar{B}^0 \to \pi^\pm\rho^\mp$	$28.08^{+0.27}_{-0.19}{}^{+3.82}_{-3.50}$	$21.90^{+0.20}_{-0.12}{}^{+3.06}_{-3.55}$	23.0 ± 2.3
$\bar{B}^0 \to \pi^0\rho^0$	$0.52^{+0.04}_{-0.03}{}^{+1.11}_{-0.43}$	$1.49^{+0.07}_{-0.07}{}^{+1.77}_{-1.29}$	2.0 ± 0.5
$B^- \to \rho_L^-\rho_L^0$	$18.42^{+0.23}_{-0.21}{}^{+3.92}_{-2.55}$	$19.06^{+0.24}_{-0.22}{}^{+4.59}_{-4.22}$	$22.8^{+1.8}_{-1.9}$
$\bar{B}_d^0 \to \rho_L^+\rho_L^-$	$25.98^{+0.85}_{-0.77}{}^{+2.93}_{-3.43}$	$20.66^{+0.68}_{-0.62}{}^{+2.99}_{-3.75}$	$23.7^{+3.1}_{-3.2}$
$\bar{B}_d^0 \to \rho_L^0\rho_L^0$	$0.39^{+0.03}_{-0.03}{}^{+0.83}_{-0.36}$	$1.05^{+0.05}_{-0.04}{}^{+1.62}_{-1.04}$	$0.55^{+0.22}_{-0.24}$

$f_+^{B\pi}(0) = 0.23 \pm 0.03$, $A_0^{B\rho}(0) = 0.28 \pm 0.03$, $\lambda_B(1\,\text{GeV}) = (0.20^{+0.05}_{-0.00})\,\text{GeV}$.

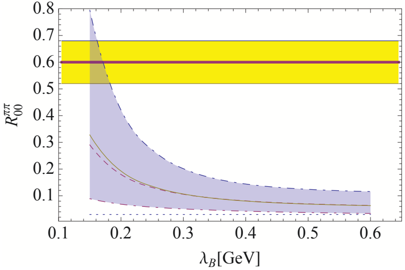
Color-suppressed versus color-allowed decays, I

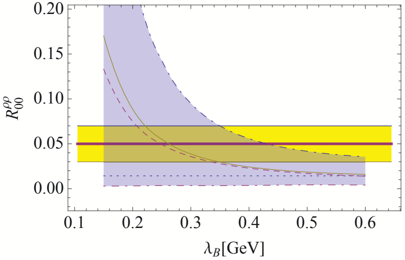


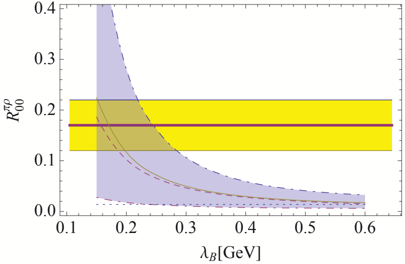
$$R_{+-}^{\pi\pi} \equiv 2\,\frac{\Gamma(B^- \to \pi^-\pi^0)}{\Gamma(B^0 \to \pi^+\pi^-)}$$

- NF fails to describe the data obviously;
- QCDF could describe the data, especially with a smaller λ_B;

Color-suppressed versus color-allowed decays, II



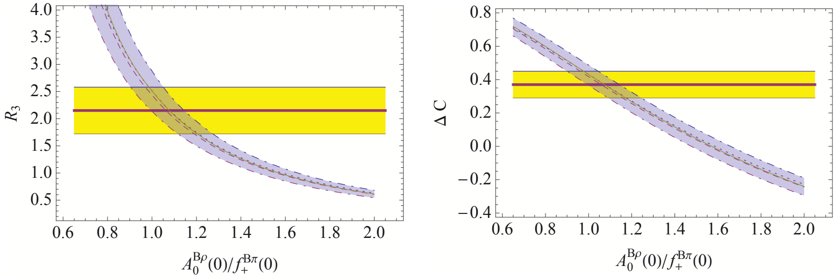




- $R_{00}^{\pi\pi} \equiv 2\,\frac{\Gamma(B^0\to\pi^0\pi^0)}{\Gamma(B^0\to\pi^+\pi^-)}$.
- prefer to smaller λ_B.
- strong spectator-scattering effects!

Charged $\pi^{\mp}\rho^{\pm}$ decay modes

$$R_3 \equiv \frac{\Gamma(\bar{B}^0 \to \pi^+\rho^-)}{\Gamma(\bar{B}^0 \to \pi^-\rho^+)}\,, \qquad \Delta C \equiv \frac{1}{2}\left[C(\pi^-\rho^+) - C(\pi^+\rho^-)\right]\,,$$



- Favor slightly smaller $B \to \rho$ to $B \to \pi$ form factor ratio than QCDSR (≈ 1.25).

Motivation for the NNLO matching calculation

$\bar{q}\,\Gamma_i\,b$, with $\Gamma_i = \{1, \gamma_5, \gamma^\mu, \gamma_5\gamma^\mu, i\sigma^{\mu\nu}\}$, play an important role in B physics:

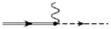
- govern the hadronic dynamics in inclusive semi-leptonic and radiative B decays: $B \to X_s\gamma$, $B \to X_u\ell\nu_\ell$, $B \to X_s\ell^+\ell^-$,...
- its matrix elements$\Longrightarrow$form factors, also important inputs to exclusive B decay processes;
- in the kinematic region: *the FS has small invariant mass but large energy* $\Longrightarrow$SCET is the appropriate theoretical framework, transparent factorization formulae can be derived;

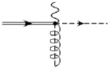
$\Longrightarrow$*how to accurately represent the heavy-to-light currents in SCET is of particular interest!*

Heavy-to-light currents in SCET

2-body operators 3-body operators

$$\bar{q}\,\Gamma b = \sum_i \int ds\; \tilde{C}_i^{(A)}(s)\; O_i^{(A)}(s) + \sum_i \int ds_1 ds_2\; \tilde{C}_i^{(B)}(s_1,s_2)\; O_i^{(B)}(s_1,s_2) + \ldots$$





- $C_i^{(A)}$ and $C_i^{(B)}$: hard coefficients;

  1-loop result is well-known,

  [*Bauer,Fleming,Pirjol,Stewart 00, Beneke,Kiyo,Yang 04; Becher, Hill 04*]

  -2-loop result: known only for (V-A) current,

  [*Bell'07,'08; Bonciani,Ferroglia'08; Asatrian,Greub,Pecjak'08; Beneke,Huber,Li'08*]

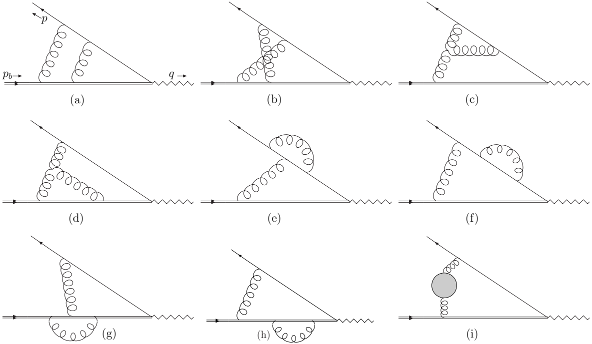
- Inclusive processes: $\langle O_i^{(A)} \rangle \to J \otimes S$, $\quad \langle O_i^{(B)} \rangle \to \sum_i j_i \otimes s_i$;

- Exclusive processes: $\langle O_i^{(A)} \rangle \to$soft-overlap contribution,
  $\langle O_i^{(B)} \rangle \to$hard spectator-scattering.

Relevant Feynman diagrams in QCD

- in the leading power:

$$[\bar{q}\,\Gamma_i\, b](0) \simeq \sum_j \int ds\; \tilde{C}_i^j(s)\,\left[\bar{\xi} W_{hc}\right](s n_+)\,\Gamma_j'\, h_v(0)$$



- here there are less diagrams and no evanescent operators;

UV renormalization and IR subtraction

- for UV renormalization, needs standard QCD counterterm:
  - the heavy quark mass;
  - the heavy and light quark field; [*Gray, Broadhurst, Grafe, Schilcher 90*]
  - the QCD current renormalization constant; [*Tarrach 81; Broadhurst, Grozin 94*]

- for IR subtraction, remember $C_i C_j * J \otimes S$ should be finite:
  - needs the SCET current renormalization constant;

$$Z_J = 1 + \frac{\alpha_s^{(4)} C_F}{4\pi} \left\{ -\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \left[-\ln\left(\frac{\mu^2}{u^2 m_b^2}\right) - \frac{5}{2} \right] \right\} + \mathcal{O}(\alpha_s^2)$$

  - 1-loop result known; [*Bauer, Fleming, Pirjol, Stewart 00*]
  - 2-loop extracted from $J \otimes S$; [*Becher, Neubert 05,06*]

Matching coefficients C_i^j for various Dirac structures

Γ_i	1	γ_5	γ^μ			$\gamma_5\gamma^\mu$			$i\sigma^{\mu\nu}$			
Γ'_j	1	γ_5	γ^μ	v^μ	n_-^μ	$\gamma_5\gamma^\mu$	$v^\mu\gamma_5$	$n_-^\mu\gamma_5$	$\gamma^{[\mu}\gamma^{\nu]}$	$v^{[\mu}\gamma^{\nu]}$	$n_-^{[\mu}\gamma^{\nu]}$	$n_-^{[\mu}v^{\nu]}$
C_i^j	C_S	C_P	C_V^1	C_V^2	C_V^3	C_A^1	C_A^2	C_A^3	C_T^1	C_T^2	C_T^3	C_T^4

- $\Gamma = \gamma^\mu$: relevant to $B \to X_u \ell \nu_\ell$, $B \to \pi \ell \nu_\ell$;
- $\Gamma = 1, \sigma^{\mu\nu}$: relevant to $B \to X_s \ell^+ \ell^-$, $B \to K^{(*)} \ell^+ \ell^-$;
- many applications to B decays:

  -V_{ub} determination from $B \to X_u \ell \nu_\ell$, [*Greub, Neubert and Pecjak 09*]

  -transition form factor ratios, [*Beneke and Feldmann 00, 03; Beneke and Yang 05*]

  -FBA in inclusive and exclusive $B \to X_s \ell^+ \ell^-$ decays,
  [*Lee, Ligeti, Stewart and Tackmann 06,07; Lee, Ligeti, Tackmann 08*]

- details to be found in

Bell, Beneke, Huber, Li, "Matching heavy-to-light current to NNLO in SCET"

Conclusion and outlook

- Overview of current NNLO corrections to hadronic B decays:
  - 1-loop spectator-scattering now complete;
  - 2-loop vertex corrections to topological tree amplitudes almost complete;
- Factorization shown to be hold in a very non-trivial way:
  - both UV- and IR- divergences cancel;
  - convolution integrations with LCDAs are finite;
  - perturbative theory well behaved at hard $\mu_h = m_b$ and hard-collinear $\mu_{hc} = \sqrt{\Lambda_{\rm QCD} m_b}$ scales;
- individual NNLO contributions are sizeable, while overall contributions only moderate; but may change the overall pattern of CP asymmetries;
- To do next: *NNLO corrections to topological penguin amplitudes*

Backup slides

Some comments on the QCD factorization approach

- *Successes*: systematic framework to calculate $\langle M_1 M_2 | Q_i | \bar{B} \rangle$; very successful:
  - consistent global description of a variety of decay modes;
  - strong phases start at $\mathcal{O}(\alpha_s)$, dynamical explanation of smallness of direct CP asymmetries in hadronic B decays;
  - dynamical explanation of the BR pattern of $B \to \eta^{(\prime)} K^{(*)}$;
  - dynamical explanation of intricate pattern of penguin interference seen in PP, PV and VP modes;

- *Limitations*: some issues still remain to be resolved:
  - factorization of power corrections is generally broken;
  - endpoint divergences in higher-twist spectator-scattering and annihilation contributions appear, bringing large model-dependent uncertainties;
  - could not account for some data, such as large $BR(B \to \pi^0 \pi^0)$, the unmatched CP asymmetries in $B \to \pi K$ decays....

Operator basis in QCD

- adopt the CMM operator basis in our calculation;
- need to introduce four extra evanescent operators;

$$Q_1 = \left[\bar{u}\gamma^{\mu}L\,T^A b\right]\ \left[\bar{d}\gamma_{\mu}L\,T^A u\right],$$

$$Q_2 = \left[\bar{u}\gamma^{\mu}L\,b\right]\ \left[\bar{d}\gamma_{\mu}L\,u\right],$$

$$E_1 = \left[\bar{u}\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}L\,T^A b\right]\ \left[\bar{d}\gamma_{\mu}\gamma_{\nu}\gamma_{\rho}L\,T^A u\right] - 16\,Q_1,$$

$$E_2 = \left[\bar{u}\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}L\,b\right]\ \left[\bar{d}\gamma_{\mu}\gamma_{\nu}\gamma_{\rho}L\,u\right] - 16\,Q_2,$$

$$E_1' = \left[\bar{u}\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma}\gamma^{\tau}L\,T^A b\right]\ \left[\bar{d}\gamma_{\mu}\gamma_{\nu}\gamma_{\rho}\gamma_{\sigma}\gamma_{\tau}\,L\,T^A u\right] - 20\,E_1 - 256\,Q_1,$$
$$E_2' = \left[\bar{u}\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma}\gamma^{\tau}L\,b\right]\ \left[\bar{d}\gamma_{\mu}\gamma_{\nu}\gamma_{\rho}\gamma_{\sigma}\gamma_{\tau}\,L\,u\right] - 20\,E_2 - 256\,Q_2,$$

$\mathrm{SCET_I}(hc, c, s)$ operator basis

- convention in SCET: M_1, along n_-, $\not{n}_-\xi = 0$; M_2, along n_+, $\not{n}_+\chi = 0$; h_v, with $\not{v}h_v = h_v$.
- Right insertion:

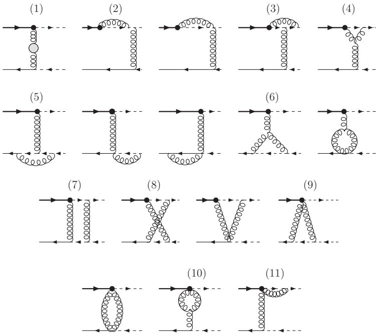
$$
\begin{aligned}
O_1 &= \left[\bar{\chi}\,\frac{\not{n}_-}{2}(1-\gamma_5)\chi\right]\left[\bar{\xi}\,\not{n}_+(1-\gamma_5)h_v\right],\\
O_2 &= \left[\bar{\chi}\,\frac{\not{n}_-}{2}(1-\gamma_5)\gamma_\perp^\alpha\gamma_\perp^\beta\chi\right]\left[\bar{\xi}\,\not{n}_+(1-\gamma_5)\gamma_\beta^\perp\gamma_\alpha^\perp h_v\right],\\
O_3 &= \left[\bar{\chi}\,\frac{\not{n}_-}{2}(1-\gamma_5)\gamma_\perp^\alpha\gamma_\perp^\beta\gamma_\perp^\gamma\gamma_\perp^\delta\chi\right]\left[\bar{\xi}\,\not{n}_+(1-\gamma_5)\gamma_\delta^\perp\gamma_\gamma^\perp\gamma_\beta^\perp\gamma_\alpha^\perp h_v\right].
\end{aligned}
$$

- Wrong insertion:

$$
\begin{aligned}
\tilde{O}_1 &= \left[\bar{\xi}\,\gamma_\perp^\alpha(1-\gamma_5)\chi\right]\left[\bar{\chi}(1+\gamma_5)\gamma_\alpha^\perp h_v\right],\\
\tilde{O}_2 &= \left[\bar{\xi}\,\gamma_\perp^\alpha\gamma_\perp^\beta\gamma_\perp^\gamma(1-\gamma_5)\chi\right]\left[\bar{\chi}(1+\gamma_5)\gamma_\alpha^\perp\gamma_\gamma^\perp\gamma_\beta^\perp h_v\right],\\
\tilde{O}_3 &= \left[\bar{\xi}\,\gamma_\perp^\alpha\gamma_\perp^\beta\gamma_\perp^\gamma\gamma_\perp^\delta\gamma_\perp^\epsilon(1-\gamma_5)\chi\right]\left[\bar{\chi}(1+\gamma_5)\gamma_\alpha^\perp\gamma_\epsilon^\perp\gamma_\delta^\perp\gamma_\gamma^\perp\gamma_\beta^\perp h_v\right].
\end{aligned}
$$

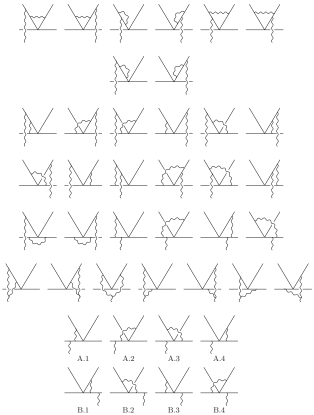
1-loop Jet function calculation

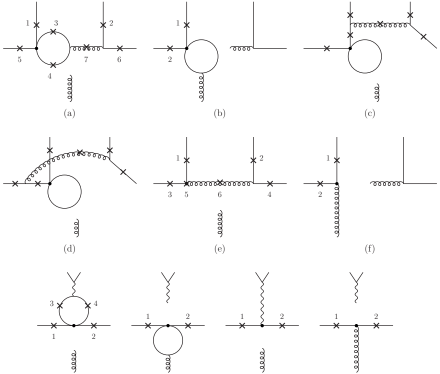
- J: now known to NLO [*Beneke,Yang 05; Becher,Hill,Lee,Neubert 04*]
- just the SD coefficient from hard-collinear scale μ_{hc};
- even at the hard-collinear scale, PT is still well-behaved.



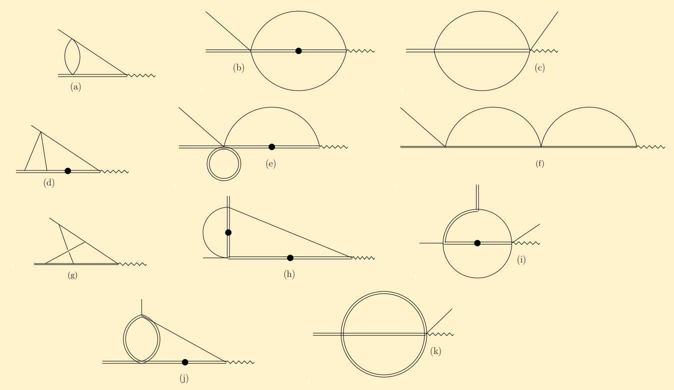
1-loop calculation for H^{II}

- diagrammatical approach (QCDF) [*Pilipp 07*]
  $\rightleftharpoons$ the effective field theory formulation (SCET) [*Beneke,Jager 05; Kivel 06*];
- factorization does hold at μ_{hc}; PT is well-behaved;



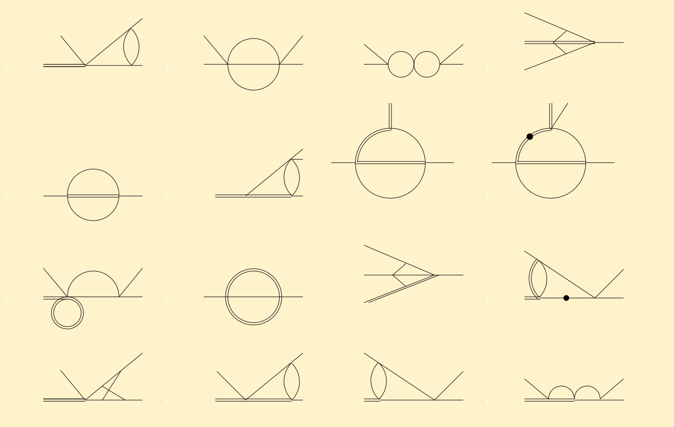


List of Master Integrals, I



- double lines are massive, while single lines massless;
- dots on lines denote squared propagators;
- have been cross-checked in $B \to X_u l\nu$ calculation, [*Bell'07,'08; Bonciani,Ferroglia'08; Asatrian,Greub,Pecjak'08; Beneke,Huber,Li'08*]

List of Master Integrals, II



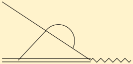
- these are extra MIs needed for hadronic B decays;
- have also been cross-checked, [*Bell'07,'09; Beneke,Huber,Li'09*]

Illustration of the techniques, I

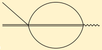
- IBP IDs: for 2-loop case, 8 IDs per scalar integral

$$\int\frac{d^Dk}{(2\pi)^D}\int\frac{d^Dl}{(2\pi)^D}\,\frac{\partial}{\partial a^\mu}\left[b^\mu f(k,l,p_i)\right]=0\,;\quad a^\mu=k^\mu,\,l^\mu\,;\quad b^\mu=k^\mu,\,l^\mu,\,p_i^\mu$$

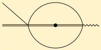
- Solve systems of these equations via Laporta algorithm
  $\Longrightarrow$scalar integrals can be expressed as a linear combination of MIs:



$$=\frac{(8-3D)(7uD-8D-24u+28)}{3(D-4)^2\,m_b^4\,u^3}$$



$$-\frac{2[u^2(D-4)+(16D-56)(1-u)]}{3(D-4)^2\,m_b^2\,u^3}$$



- Differential equations: [*Kotikov'91; Remiddi'97*]

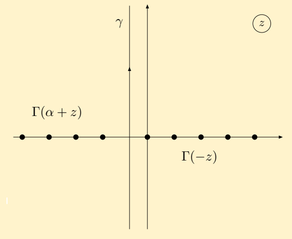
$$\frac{\partial}{\partial u}\mathrm{MI}_i(u)=f(u,\epsilon)\,\mathrm{MI}_i(u)+\sum_{j\neq i}g_j(u,\epsilon)\,\mathrm{MI}_j(u)$$

- needs result from Laporta reduction.
- boundary condition from some other techniques.

Illustration of the techniques, II

- Mellin-Barnes representation: [*Smirnov'99; Tausk'99*]

$$\frac{1}{(A_1+A_2)^\alpha} = \oint_\gamma \frac{dz}{2\pi i} A_1^z A_2^{-\alpha-z} \frac{\Gamma(-z)\Gamma(\alpha+z)}{\Gamma(\alpha)}$$

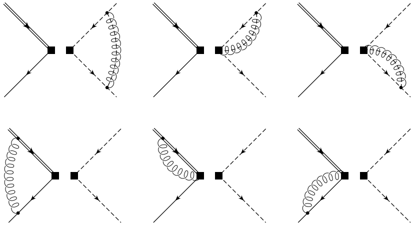


- partially automated,
- used as numerical cross checks of our analytic calculation, [*Czakon'05; Gluza,Kajda,Riemann'07*]

- Special functions frequently used:
- HPL up to weight 4 with argument u or $1-u$
- Polylogarithms Li_2, Li_3, Li_4 with argument u, $1-u$, $\frac{u}{1-u}$
- Hypergeometric function pFq, ϵ-expansion, [*Maitre,Huber'05,'07*]

Subtleties in performing IR-subtraction

- To prove factorization, should check that $T^{I,II}$ are free of both UV- and IR-divergences, *very non-trivial both in QCD and in SCET!*
- should consider the evanescent operator contributions from QCD and SCET;
- some operators are non-local, needs distribution like ERBL kernel up to two-loop order, [*Lepage,Brodsky'80; Efremov,Radyushkin'80*]
- SCET graphs contributing to the anomalous dimension Y_{ij} of the SCET four-quark operators:



Dependence of $\alpha_{1,2}$ on the Gegenbauer moments.

- at LO, no such a dependence at all in QCDF/SCET;
- at NLO, the dependence is ($\mu = m_b$):

$$
\begin{aligned}
[\alpha_1]_V &= 1.040 + 0.013i - (0.007 - 0.013i)\, a_1^{M_2} + 0.001\, a_2^{M_2} \\
[\alpha_2]_V &= 0.029 - 0.079i + (0.046 - 0.079i)\, a_1^{M_2} - 0.009\, a_2^{M_2}
\end{aligned}
$$

- at NNLO, the dependence is ($\mu = m_b$):

$$
\begin{aligned}
[\alpha_1]_V &= 1.057 + 0.038i - (0.032 - 0.022i)\, a_1^{M_2} + (0.003 - 0.001i)\, a_2^{M_2}, \\
[\alpha_2]_V &= 0.013 - 0.126i + (0.139 - 0.096i)\, a_1^{M_2} - (0.021 + 0.009i)\, a_2^{M_2}.
\end{aligned}
$$

- the dependence on $a_2^{M_2}$ is very small for both tree amplitudes;
- the first Gegenbauer moment $a_1^{M_2}$ is more important, especially for $[\alpha_2]_V$, furthermore enhanced at NNLO;
- a source of non-factorizable SU(3) flavour symmetry breaking.

Definition of various ratios in tree-dominated decays, I

$$R_{+-}^{\pi\pi} \equiv 2\,\frac{\Gamma(B^- \to \pi^-\pi^0)}{\Gamma(B^0 \to \pi^+\pi^-)}\,, \qquad R_{00}^{\pi\pi} \equiv 2\,\frac{\Gamma(B^0 \to \pi^0\pi^0)}{\Gamma(B^0 \to \pi^+\pi^-)}\,,$$

$$R_{+-}^{\rho\rho} \equiv 2\,\frac{\Gamma(B^- \to \rho_L^-\rho_L^0)}{\Gamma(B^0 \to \rho_L^+\rho_L^-)}\,, \qquad R_{00}^{\rho\rho} \equiv 2\,\frac{\Gamma(B^0 \to \rho_L^0\rho_L^0)}{\Gamma(B^0 \to \rho_L^+\rho_L^-)}\,,$$

$$R_{00}^{\pi\rho} \equiv \frac{2\,\Gamma(B^0 \to \pi^0\rho^0)}{\Gamma(B^0 \to \pi^+\rho^-) + \Gamma(B^0 \to \pi^-\rho^+)}\,.$$

- the dependence on FF and CKM factor cancels out;
- $R^{+-} \propto |\,\alpha_1 + \alpha_2\,|^2/|\,\alpha_1\,|^2$, while $R^{00} \propto |\,\alpha_2\,|^2/|\,\alpha_1\,|^2$
- highlight the importance of hard spectator-scattering;
- strongly depend on the B-meson LCDA λ_B and $r_{\rm sp} = \frac{9 f_{M_2}\hat{f}_B}{m_b F F^{BM_1}(0)\lambda_B}$;

Definition of various ratios in tree-dominated decays, II

$$R_1 \equiv \frac{\Gamma(\bar{B}^0 \to \pi^+\rho^-)}{\Gamma(\bar{B}^0 \to \pi^+\pi^-)}, \qquad R_2 \equiv \frac{\Gamma(\bar{B}^0 \to \pi^+\rho^-) + \Gamma(\bar{B}^0 \to \pi^-\rho^+)}{2\,\Gamma(\bar{B}^0 \to \pi^+\pi^-)},$$

$$R_3 \equiv \frac{\Gamma(\bar{B}^0 \to \pi^+\rho^-)}{\Gamma(\bar{B}^0 \to \pi^-\rho^+)}, \qquad R_4 \equiv \frac{2\,\Gamma(B^- \to \pi^-\rho^0)}{\Gamma(\bar{B}^0 \to \pi^-\rho^+)} - 1,$$

$$R_5 \equiv \frac{2\,\Gamma(B^- \to \pi^0\rho^-)}{\Gamma(\bar{B}^0 \to \pi^+\rho^-)} - 1, \qquad R_6 \equiv \frac{\Gamma(\bar{B}^0 \to \pi^+\rho^-) + \Gamma(\bar{B}^0 \to \pi^-\rho^+)}{2\,\Gamma(\bar{B}^0 \to \rho_L^+\rho_L^-)}.$$

$$R_C^\pi = \frac{\Gamma(\bar{B}^0 \to \pi^0\pi^0)}{\Gamma(\bar{B}^0 \to \pi^0\rho^0)}, \qquad R_C^\rho = \frac{\Gamma(\bar{B}^0 \to \rho_L^0\rho_L^0)}{\Gamma(\bar{B}^0 \to \pi^0\rho^0)}.$$

- R_1 is free of FF dependence; $R_3 \propto A_0^{B\rho}(0)/f_+^{B\pi}(0)$;
- $R_C^{\pi,\rho}$ provide information on α_2 in PP, PV and VV modes;
- $R_{4,5,6}$ is almost free of CKM factor dependence;
- $R_{4,5}$ provide access to the real part of $\alpha_2(\pi\rho)$ and $\alpha_2(\rho\pi)$;

Various ratios of $B \to \pi\pi$, $\pi\rho$ and $\rho_L\rho_L$ decays

	Theory I	Theory II	Experiment
$R^{\pi\pi}_{+-}$	$1.38^{+0.12+0.53}_{-0.13-0.32}$	$1.91^{+0.18+0.72}_{-0.20-0.64}$	2.02 ± 0.17
$R^{\pi\pi}_{00}$	$0.09^{+0.03+0.12}_{-0.02-0.04}$	$0.22^{+0.06+0.28}_{-0.05-0.16}$	0.60 ± 0.08
$R^{\rho\rho}_{+-}$	$1.32^{+0.02+0.44}_{-0.03-0.27}$	$1.72^{+0.03+0.64}_{-0.03-0.53}$	$1.80^{+0.28}_{-0.29}$
$R^{\rho\rho}_{00}$	$0.03^{+0.00+0.07}_{-0.00-0.03}$	$0.10^{+0.01+0.19}_{-0.01-0.11}$	0.05 ± 0.02
$R^{\pi\rho}_{00}$	$0.04^{+0.00+0.09}_{-0.00-0.03}$	$0.14^{+0.01+0.20}_{-0.01-0.13}$	0.17 ± 0.05
R_1	$2.41^{+0.16+0.32}_{-0.18-0.37}$	$2.41^{+0.17+0.37}_{-0.20-0.43}$	3.04 ± 0.37
R_2	$1.90^{+0.18+0.53}_{-0.19-0.41}$	$1.92^{+0.19+0.42}_{-0.20-0.40}$	2.23 ± 0.24
R_3	$1.73^{+0.13+1.12}_{-0.12-0.82}$	$1.69^{+0.13+0.72}_{-0.12-0.59}$	2.15 ± 0.43
R_4	$0.58^{+0.02+0.67}_{-0.02-0.35}$	$1.26^{+0.00+0.84}_{-0.00-0.75}$	$1.12^{+0.46}_{-0.48}$
R_5	$0.30^{+0.05+0.36}_{-0.04-0.20}$	$0.64^{+0.06+0.50}_{-0.05-0.41}$	$0.30^{+0.22}_{-0.23}$
R_6	$0.54^{+0.01+0.23}_{-0.01-0.17}$	$0.53^{+0.01+0.16}_{-0.01-0.13}$	0.49 ± 0.08
R^{π}_{C}	$0.64^{+0.22+0.64}_{-0.17-0.37}$	$0.42^{+0.09+0.28}_{-0.08-0.16}$	0.78 ± 0.22
R^{ρ}_{C}	$0.74^{+0.10+0.58}_{-0.09-0.46}$	$0.70^{+0.06+0.46}_{-0.06-0.39}$	$0.27^{+0.13}_{-0.14}$

Input parameters used in the phenomenological analysis

Parameter	Value/Range	Parameter	Value/Range
$\Lambda^{(5)}_{\overline{\rm MS}}$	0.225	$\mu_{\rm hc}$	1.5 ± 0.6
m_c	1.3 ± 0.2	f_{B_d}	0.195 ± 0.015
$m_s(2\text{ GeV})$	0.09 ± 0.02	f_π	0.131
$(m_u + m_d)/m_s$	0.0826	$f_+^{B\pi}(0)$	$0.25 \pm 0.05^\dagger$
m_b	4.8	f_ρ	0.209
$\bar{m}_b(\bar{m}_b)$	4.2	$A_0^{B\rho}(0)$	$0.30 \pm 0.05^\dagger$
$\lvert V_{cb}\rvert$	0.0415 ± 0.0010	$\lambda_B(1\text{ GeV})$	$0.35 \pm 0.15^\dagger$
$\lvert V_{ub}/V_{cb}\rvert$	0.09 ± 0.02	$\sigma_1(1\text{ GeV})$	1.5 ± 1
γ	$(70 \pm 10)^\circ$	$\sigma_2(1\text{ GeV})$	3 ± 2
$\tau(B^-)$	$1.64\,\text{ps}$	$a_2^\pi(2\text{ GeV})$	0.2 ± 0.15
$\tau(B_d)$	$1.53\,\text{ps}$	$a_2^\rho(2\text{ GeV})$	0.1 ± 0.15
μ_b	$4.8^{+4.8}_{-2.4}$	$a_{2,\perp}^\rho(2\text{ GeV})$	0.1 ± 0.15

† $f_+^{B\pi}(0) = 0.23 \pm 0.03$, $A_0^{B\rho}(0) = 0.28 \pm 0.03$, $\lambda_B(1\,\text{GeV}) = (0.20^{+0.05}_{-0.00})$ GeV.