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The light Scalar Mesons: σ and κ

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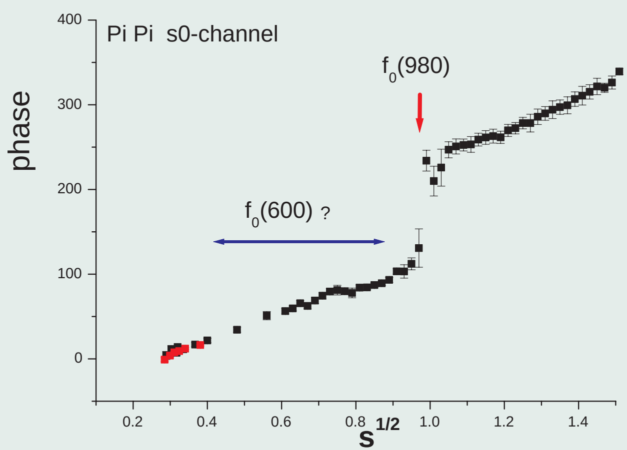
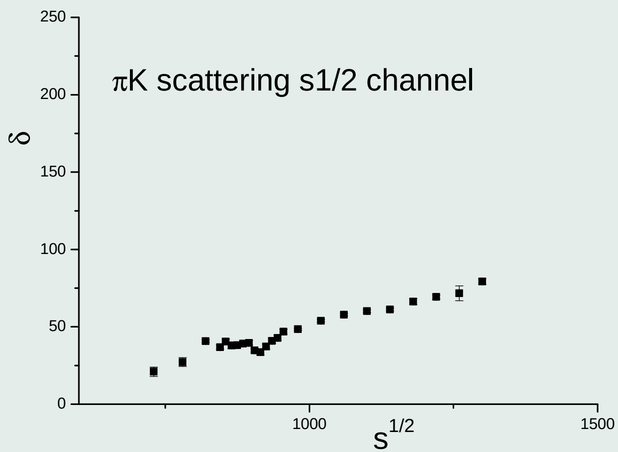
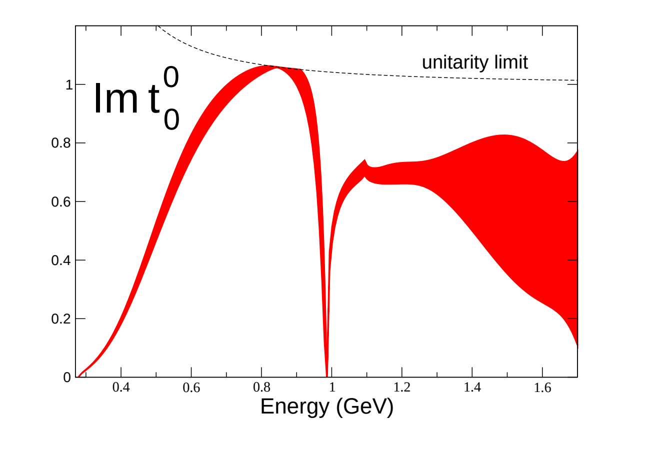
1 Background

Linear σ model→Nonlinear σ model

σ particle: appear → disappear → reappear in PDG







There is the broad object seen in $\pi\pi$ *scattering, often called "background", which extends from about 400 MeV up to about 1700 MeV. This object we consider as a single broad resonance*[2] *which we identify as the lightest glueball with quantum numbers* $J^{PC} = 0^{++} \ldots$

[2] *we refer to it as red dragon*

P. Minkowski and W. Ochs, Eur. Phys. J. C9 (1999) 283

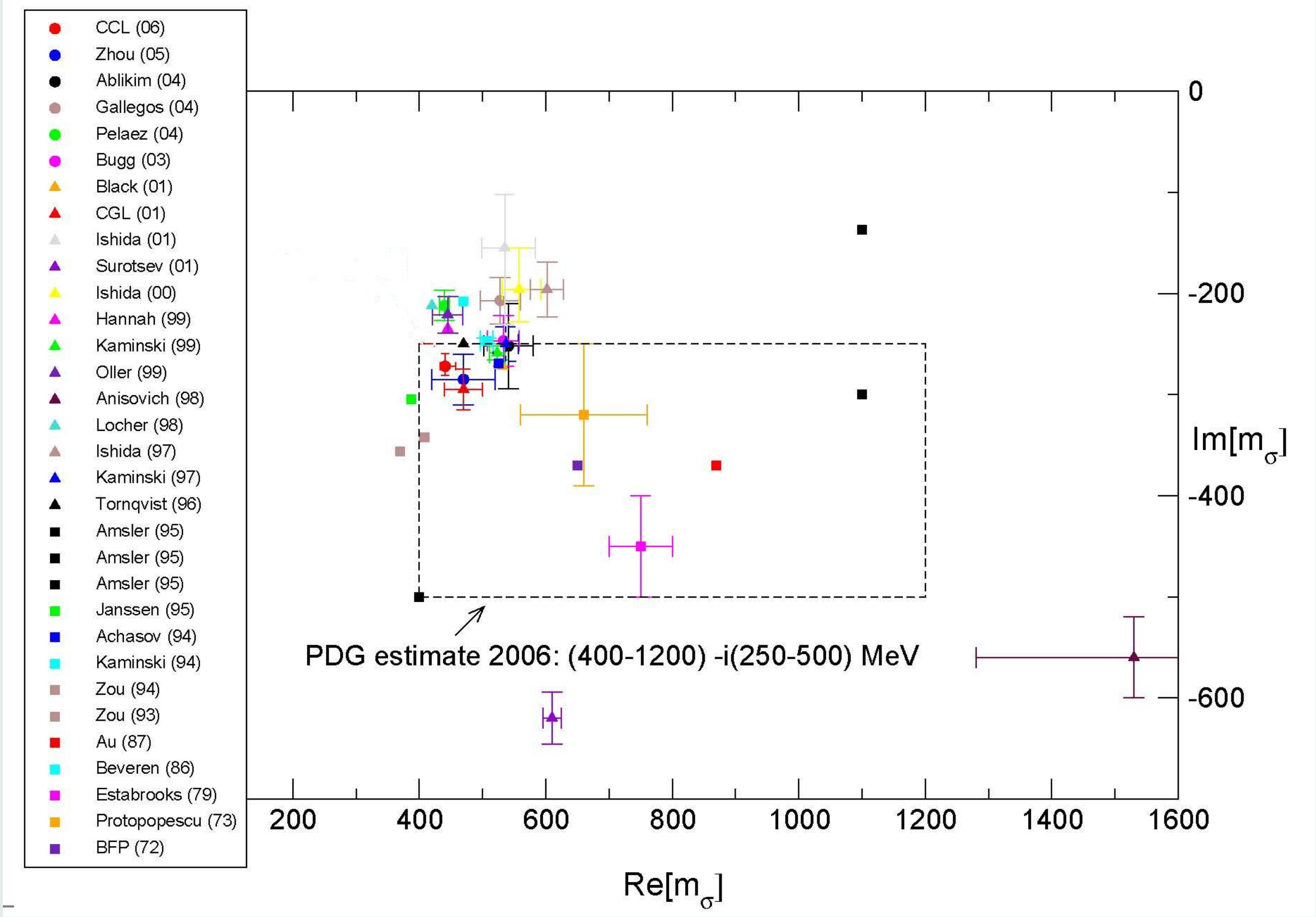
The Red Dragon

- Does QCD have a resonance near threshold ?
- Why care ?
  - Concerns the nonperturbative domain of QCD
  - Quark and gluon degrees of freedom useless there

  $\Rightarrow$ Understanding very poor, pattern of energy levels ?
  - Lowest resonance: σ ? ρ ?
- Resonance $\leftrightarrow$ pole on second sheet
  - Poles are universal
  - Pole position is unambiguous, even if width is large
  - Where is the pole closest to the origin ?

as pointed by H.Leutwyler in QCHS VII

Comparison with compilation of PDG



Different methods in studying $\pi\pi$ physics

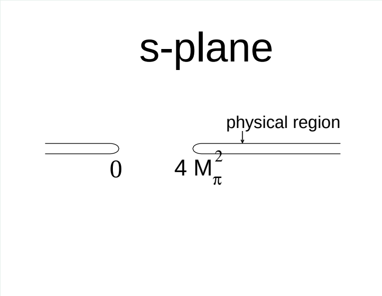
1. Chiral Perturbation Theory(without the information of resonance)

2. Phenomenological Analysis(model-dependent, hardly to be trusted in studying broad resonance)

3. Dispersive technique(including Roy equation method, S matrix theory...), model-independent

2 Analytical Continuation

pole on second sheet $\longleftrightarrow$ zero on first sheet



- $S_0^0(s) = \eta_0^0(s)\exp 2i\delta_0^0(s)$

  $S_0^0(s)$ is analytic in the cut plane

- For $0 < s < 4M_\pi^2$, $S_0^0(s)$ is real

$\Rightarrow$ $S_0^0(s^\star) = S_0^0(s)^\star$

x in elastic interval: $S_0^0(x \pm i\epsilon) = \exp \pm 2i\delta_0^0(x)$

- Second sheet is reached by continuation across the elastic interval of the right hand cut

  $S_0^0(x - i\epsilon)^{II} = S_0^0(x + i\epsilon)^{I} = 1/S_0^0(x - i\epsilon)^{I}$

  Analyticity $\Rightarrow$ $\boxed{S_0^0(s)^{II} = 1/S_0^0(s)^{I}}$ valid $\forall s$

  Pole in $S_0^0(s)^{II}$ $\Longleftrightarrow$ zero in $S_0^0(s)^{I}$

3 PKU Parametrization Form

The most important characters in studying low energy physics

1. Unitarity
2. Crossing symmetry
3. Analyticity

$$S^{phy} = S^{cut} \cdot \prod_i S_i^p \,, \tag{1}$$

S^{cut} is expressed based on dispersion relation:

$$S^{cut} = \exp[2i\rho(s)f(s)] \,, \tag{2}$$

$$\begin{aligned} f(s) &= f(s_0) + \frac{(s-s_0)}{2\pi i}\int_L \frac{\mathrm{disc}_{\mathrm{L}} f(z)}{(z-s)(z-s_0)} dz \\ &+ \frac{(s-s_0)}{\pi}\int_R \frac{\mathrm{Im}_{\mathrm{R}} f(z)}{(z-s)(z-s_0)} dz \,. \end{aligned} \tag{3}$$

It conserves unitarity,

$$\text{disc} f = \text{disc}\{\frac{1}{2i\rho(s)} \log \left[S^{phy}(s)\right]\} \ . \tag{4}$$

$$\text{Im}_R f(s) = -\frac{1}{2\rho(s)} \log \eta(s) = -\frac{1}{4\rho(s)} \log(\frac{S_{11}S_{22}}{\det S}) \ , \tag{5}$$

The Simplest S Matrices

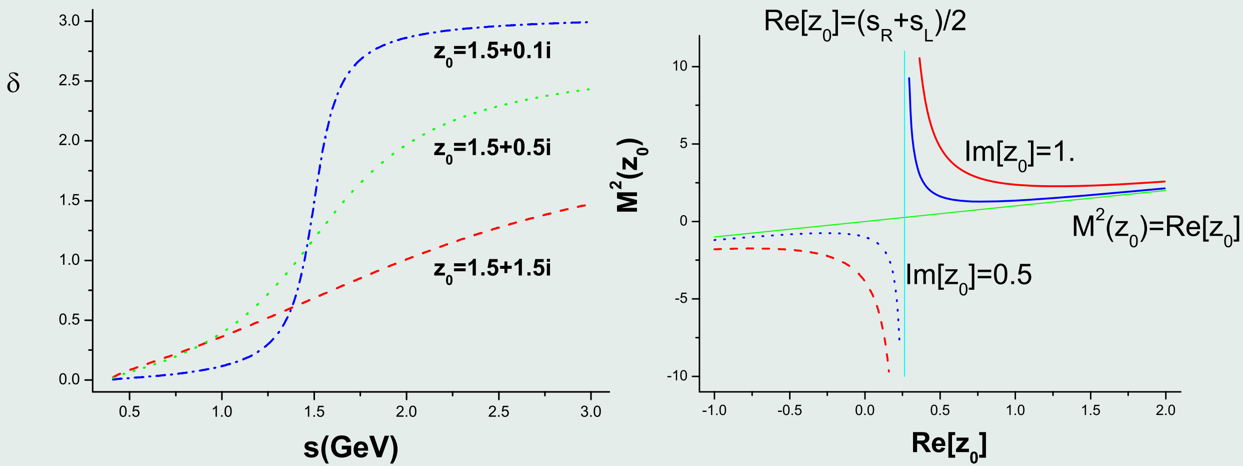
1. A virtual state → pole at s_0, with s_0 real.

$$S(s) = \frac{1 + i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_0-s_L}{s_R-s_0}}}{1 - i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_0-s_L}{s_R-s_0}}} \ . \tag{6}$$

   As for a bound state at s_0, change sign. $s_L < s_0 < s_R$.

2. A resonance → poles at z_0 and z_0^* on second sheet:

$$S(s) = \frac{M^2(z_0) - s + i\rho(s)sG}{M^2(z_0) - s - i\rho(s)sG} \ , \tag{7}$$



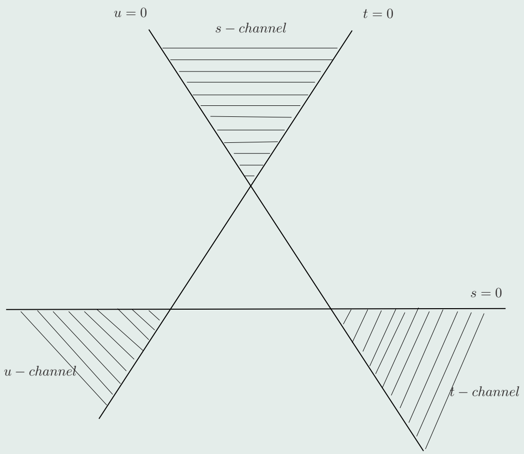
Some examples of the second sheet poles;In different $\mathrm{Im}[z_0]$, the relations of $M^2(z_0)$ and $\mathrm{Re}[z_0]$.

Recast Eq.(1) into

$$\frac{1}{2i\rho(s)}\log(1+2i\rho(s)T^{phy}(s)) = \frac{1}{2i\rho(s)}\sum_i \log(S^{R_i}(s)) + f(s)\ . \tag{8}$$

its behavious when $s \to 0_+$:

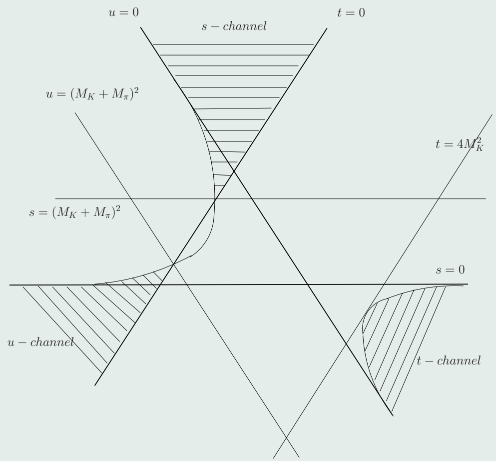
$$f(0) = 0\ , \tag{9}$$



In the case of equal-mass,

$$\begin{aligned} T^I_J(s) &= \frac{1}{32\pi(s-4m_\pi^2)}\int_{4m_\pi^2-s}^{0} dt P_J(1+\frac{2t}{s-4m_\pi^2})T^I(s,t,u)\ , \\ u &= 4m_\pi^2 - s - t \end{aligned} \tag{10}$$

$T^I_J(0_+)$ is finite based on Mandelstam analyticity.



In the case of unequal-mass, the behavior of $T(s,t)$ in $t \to \infty$ is described by $Regge Theory$. For simplicity, assume that $|T(s,t)| < |t|^n$ for a certain s and $\vee\, t$,

$$
\begin{aligned}
T_J^I(s) &= \frac{1}{32\pi}\frac{1}{2{q_s}^2}\int_{-4q_s^2}^{0} dt P_J(1+\frac{t}{2{q_s}^2})T^I(s,t,u)\ , \\
q_s^2 &= \frac{(m_K^2-m_\pi^2)^2}{4s},\ \ (s \to 0) \qquad (11)
\end{aligned}
$$

So

$$
\begin{aligned}
\lim_{s\to 0^+}|T_0^I(s)| &< \frac{1}{32\pi}\frac{1}{2{q_s}^2}\int_{-4q_s^2}^{0} dt|t|^n\ , \\
&\sim O(s^{-n}) \qquad (12)
\end{aligned}
$$

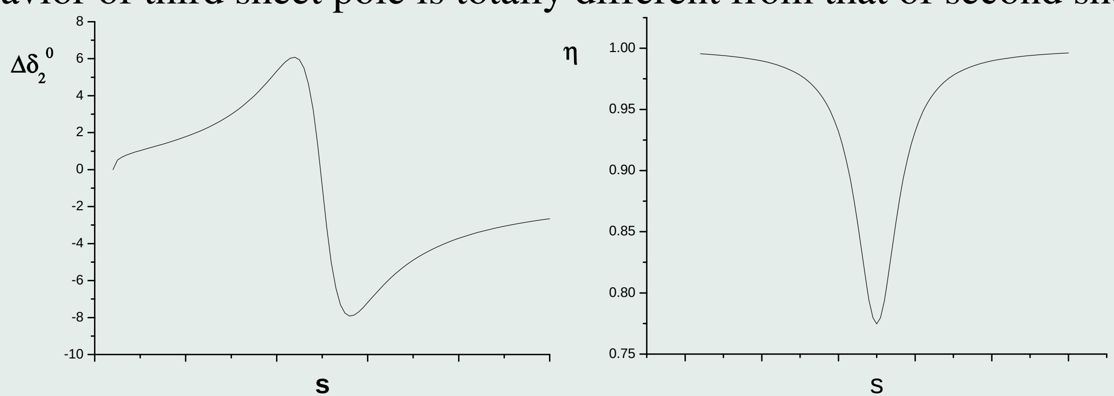
According to the multi-channel unitarity

$$\begin{aligned}\mathrm{Im}\{T_l^{ii}(s)\} &= \rho^i \mid T_l^{ii}(s) \mid^2 + \sum_{n\neq i} \rho^n T^{in}(s_+) T^{ni}(s_-) \\ &+ (3 - and\ more - body\ channels)\end{aligned} \tag{13}$$

so

$$\begin{aligned}\mathrm{Im}_R f(s) &= -\frac{1}{2\rho(s)} \log |S^{phy}(s)| \\ &= -\frac{1}{4\rho} \log \left[1 - 4\rho \mathrm{Im}_R T + 4\rho^2 |T(s)|^2\right] \\ &= -\frac{1}{4\rho} \log \left[1 - 4\rho(\sum_n \rho_n |T_{1n}(s)|^2 + \cdots)\right]\end{aligned} \tag{14}$$

The behavior of third sheet pole is totally different from that of second sheet pole:

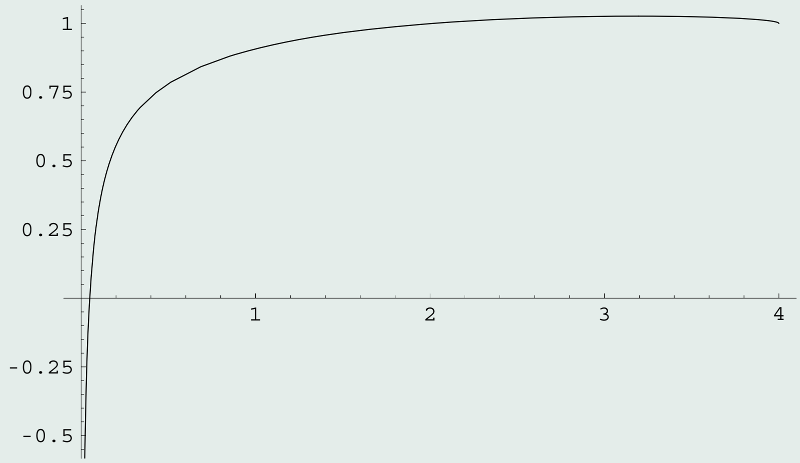


contribution of 3-sheet pole

4 $\pi\pi$ Scattering and σ resonance

Z.Y. Zhou et al.,[**JHEP 0502:043,2005**]

IJ=20 channel There exist a virtual state in $s2$ partial wave. $s_0 \simeq 0.049\mathrm{GeV}^2$, its contribution to a_0^2 is fairly large $\simeq 0.11 m_\pi^{-1}$).



IJ=11 channel

$$\mathrm{Re}T_{IJ}(s)=q^{2J}[a_J^I+b_J^Iq^2+O(q^4)]\ ,\quad (q=\frac{1}{2}\sqrt{s-4m_\pi^2}\,)\ , \tag{15}$$

leads to

$$f(4m_\pi^2)=-\sum_i a^{R_i}\ . \tag{16}$$

Thus f is subtracted at $s=0$ and $s=4m_\pi^2$,

$$\begin{aligned} f(s) &= \frac{f(4m_\pi^2)}{4m_\pi^2}s+\frac{s(s-4m_\pi^2)}{\pi}\int_{L+R}\frac{\mathrm{Im}f(s')}{s'(s'-4m_\pi^2)(s'-s)} \\ &\to -\sum_i a^{R_i}\frac{s}{4m_\pi^2}+\frac{s(s-4m_\pi^2)}{\pi}\int_{L+R}\frac{\mathrm{Im}f(s')}{s'(s'-4m_\pi^2)(s'-s)}ds'\ , \end{aligned} \tag{17}$$

.

IJ=00 channel

two resonance: σ and $f_0(980)$.

$\star$ **Crossing symmetry** $\leftarrow$ BNR relations:

$$\chi^2_{tot} = \chi^2_{00} + \chi^2_{11} + \chi^2_{20} + \chi^2_{BNR} = 29.7(36) + 214.9(39) + 41.6(23) + 4.62\ ;$$
$$M_\rho = 757.0 \pm 0.4MeV\ ,\ \ \Gamma_\rho = 152.2 \pm 0.6MeV\ ,$$
$$M_\sigma = 457 \pm 15MeV\ ,\ \ \Gamma_\sigma = 551 \pm 28MeV\ ,$$
$$(441^{+16}_{-8}MeV)\ \ (544^{+25}_{-18}MeV)\ \ \textbf{CCL, PRL96(2006)132001} \tag{18}$$

	Our results	χPT [68]	Roy Eqs. [71]	Exp. [62]	Unit
a_0^0	0.211 ± 0.011	0.220 ± 0.005	0.220 ± 0.005	0.26 ± 0.05	
b_0^0	0.264 ± 0.015	0.280 ± 0.011	0.276 ± 0.006	0.25 ± 0.03	m_π^{-2}
a_0^2	-0.440 ± 0.011	-0.423 ± 0.010	-0.444 ± 0.010	-0.28 ± 0.12	10^{-1}
b_0^2	-0.785 ± 0.010	-0.762 ± 0.021	-0.803 ± 0.012	-0.82 ± 0.08	$10^{-1}m_\pi^{-2}$
a_1^1	0.367 ± 0.003	0.380 ± 0.021	0.379 ± 0.005	0.38 ± 0.02	$10^{-1}m_\pi^{-2}$
b_1^1	0.563 ± 0.003	0.58 ± 0.12	0.567 ± 0.013		$10^{-2}m_\pi^{-4}$

表 4.1: 和其他方法所得的阈参数的结果的比较。 IJ=11 道相移数据来自 Ref. [17, 64].

5 πK Scattering and κ resonance

s-plane $K\pi$ singularity

a) s-channel unitarity cut $s \geq (M_K + M_\pi)^2$

b) u-channel unitarity cut $u \geq (M_K + M_\pi)^2$, leads to s-plane $-\infty < s \leq (M_K - M_\pi)^2$

c) t-channel unitarity cut $t \geq 4(M_\pi)^2$,leads to

- **i** circular cut $s = (M_K^2 - M_\pi^2)exp(i\phi)$
- **ii** $-\infty < s \leq 0$

d) singularity at $\mid s \mid^{-const}$

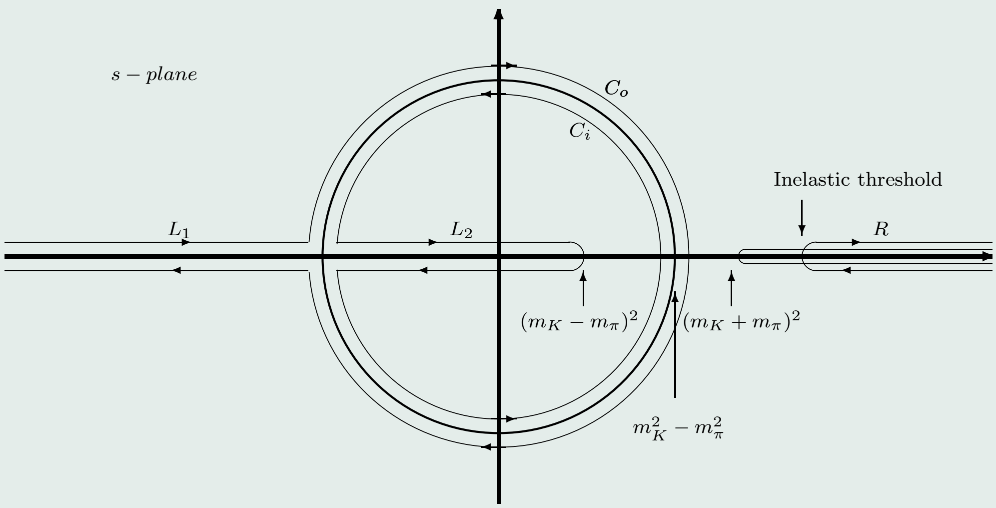
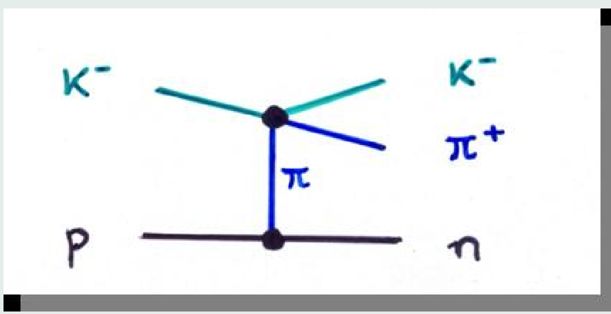
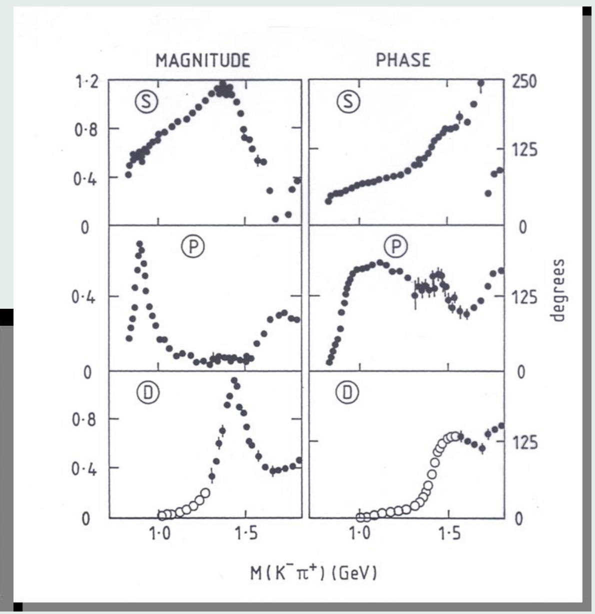


图 5.1: πK 散射的左手割线，圈割线和右手割线。

By measuring process $K^-\pi^+$, LASS Collaboration provide a combined data of a_0 and ϕ_0:

$$A_0 = a_0 e^{i\phi_0} = T_0^{1/2} + \frac{1}{2} T_0^{3/2} = \frac{1}{2i}(\eta_0^{1/2} e^{2i\delta_0^{1/2}} - 1) + \frac{1}{4i}(\eta_0^{3/2} e^{2i\delta_0^{3/2}} - 1)\,, \qquad (19)$$





Our result: $M_\sigma = 694 \pm 53 MeV\ ,\ \ \Gamma_\sigma = 606 \pm 59 MeV$

[Z. Y. Zhou, H. Q. Zheng, [Nucl. Phy. **A775** (2006) 212-223]

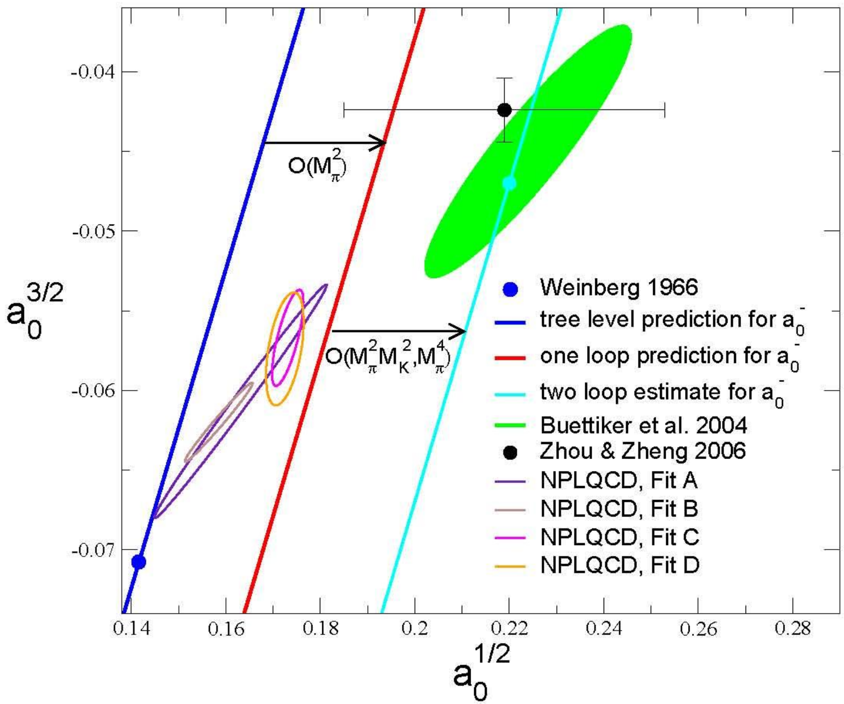
Roy-steiner relation(an analogue of Roy equation) find the lowest I=1/2, l=0 resonance sits at $M_\sigma = 658 \pm 13 MeV\ ,\ \ \Gamma_\sigma = 557 \pm 24 MeV$

[S. Descotes-Genon, B. Moussallam, hep-ph/0607133]

⇒Calculation confirms our result.

Estimate for the $O(p^6)$ couplings gives large correction

Bijnens, Dhonte & Talavera 2004, Schweizer 2005, Kaiser & Schweizer 2006



6 Conclusion

1. Including the contributions of left hand cut will stablize the low-lying pole positions in fit.
2. By a model independent approach, the existence of σ and κ is confirmed and determined accurately.

Thank you!