PARTON DISTRIBUTION FUNCTIONS AND ELECTROWEAK PHYSICS

W. J. STIRLING AND A. D. MARTIN
Institute for Particle Physics Phenomenology, University of Durham, DH1 3LE, UK

R. G. ROBERTS
Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX, UK

R. S. THORNE
Cavendish Laboratory, University of Cambridge, Cambridge, CB3 0HE, UK

We discuss two aspects of electroweak physics that relate to parton distributions: (i) the calculation of total W and Z cross sections at hadron colliders, and in particular the uncertainties in the theoretical predictions that originate in the parton distribution functions, and (ii) the effect of including $\mathcal{O}(\alpha)$ QED corrections to the parton evolution in the global analysis.

1 Introduction

Accurately determined parton distributions are an essential ingredient of precision hadron collider phenomenology. Recently the focus has been on obtaining reliable *uncertainties* on the parton distributions obtained from global or semi-global fits. One obvious uncertainty comes from the systematic and statistical errors of the data, i.e. the *experimental* errors. They have been estimated by several groups[1,2,3,4,5,6,7,8,9,10], working within a NLO (and sometimes NNLO) framework using a variety of different procedures. The general conclusion is that in all approaches the uncertainties are of the order of $\pm$ $(2-5)\%$, except for certain partons and regions of phase space (for example, gluons and sea quarks at very high x) where the uncertainty can be bigger. However, when the partons and predictions for physical quantities using different parton sets are compared, the deviations can be significantly greater than the quoted errors. This is most likely due to different assumptions made by different groups when carrying out their fitting procedures, such as differences in (i) the data sets used in the fit, (ii) the functional forms used for the input distributions, (iii) the choice of heavy flavour prescriptions, etc. However, the different results also suggest that the standard fixed order approximation in the perturbative expansion may not be completely adequate to describe the data, with different individual data sets making different and conflicting demands on the partons and on α_S. Indeed, in our (MRST) analysis we do see certain difficulties in obtaining the best possible fit to the data with the current theoretical treatment – the HERA and NMC DIS structure function data in the range $x \sim 0.01 - 0.001$ appear to increase too rapidly with Q^2; the gluon is negative (or at best valence-like) at small x and Q^2; and the Tevatron jet data prefer a different shaped gluon to the DIS data.

Hence, in order to understand the uncertainties in parton distributions, it is vital to consider also *theoretical corrections*. These include the possibility of isospin violation; $s(x) \neq \bar{s}(x)$; higher orders in perturbation theory (e.g. NNLO); QED effects (which *a priori* could be comparable to NNLO pQCD corrections, since $\alpha_s^3 \sim \alpha$); large-x and small-x logarithmic corrections, low-Q^2 power corrections, and so on. We have recently considered the uncertainties associated with many of these possible sources in Ref. [11].

The total W and Z production cross sec-

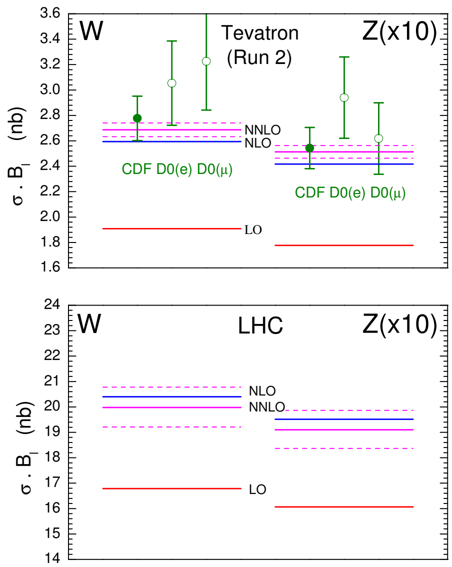

Figure 1. Predictions for the W and Z total cross sections at the Tevatron and LHC, from MRST [11], compared with recent data from CDF and D0.

tions at hadron colliders such as the Tevatron and LHC provide a useful benchmark with which to compare the parton distributions obtained in various approaches. Indeed, with sufficient theoretical precision, these cross sections could even be used as a *luminosity monitor* at these machines. The results are encouraging — Fig. 1 shows the LO, NLO and NNLO pQCD predictions from the latest MRST partons[11], together with recent CDF and D0 Tevatron cross-section measurements. The NNLO pdfs are obtained from a global analysis which utilizes the recently calculated (Moch, Vermaseren and Vogt[12]) full three-loop splitting functions. Evidently the perturbation series is well under control, and there is also good agreement between theory and experiment. Taking all sources of uncertainties in the theoretical analysis into account, we estimate a $\pm 4\%$ uncertainty in the NNLO prediction[11], shown as the horizontal dashed lines bracketing the NNLO cross sections.

LHC	$\sigma_{\rm NLO}(W)$ (nb)
MRST2002	204 $\pm$ 6
CTEQ6	205 $\pm$ 8
ALEKHIN02	215 $\pm$ 6

Table 1. W total cross section (NLO) predictions for the LHC using the same (electroweak) input parameters. The errors are defined by the various pdf 'error sets' from MRST, CTEQ and Alekhin.

However a word of caution is in order. It is important to ensure consistency between the parton distributions obtained by various groups. Table 1 shows the NLO prediction for $\sigma(W)$ at the LHC from recent MRST[10], CTEQ[6] and Alekhin[3] 'error sets'. These are designed primarily to reflect the errors on the data used in the global fits. Note that while the CTEQ and MRST central values are almost identical (the slight difference in the cross section error is due simply to a difference in the definition of the $\Delta\chi^2$ tolerance) the Alekhin central value is outside the MRST and CTEQ error band, although consistent within errors. This is most likely due to the absence of Tevatron high E_T data in the Alekhin fit[3], which allows the gluon to be larger at small x and the small-x quarks to increase correspondingly faster with Q^2. These small differences in the fitting procedures and their impact on predictions on physical quantities need to be better understood.

In the context of pQCD, the current frontier is evidently NNLO, but attention has also focused recently on electroweak radiative corrections to hadron collider cross sections. Such corrections are of course routinely applied in e^+e^- and ep collider physics, but their application to hadron colliders is relatively new. They have, for example, been discussed in the context of W, Z [13,14] and WH, ZH [15] production at hadron colliders.

QED contributions are invariably an important part of such electroweak corrections. In particular, at hadron colliders

$$\begin{aligned}
\frac{\partial q_i(x,\mu^2)}{\partial \log\mu^2} &= \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\Big\{P_{qq}(y)\; q_i(\frac{x}{y},\mu^2) + P_{qg}(y)\; g(\frac{x}{y},\mu^2)\Big\} \\
&\quad + \frac{\alpha}{2\pi}\int_x^1 \frac{dy}{y}\Big\{\tilde{P}_{qq}(y)\; e_i^2 q_i(\frac{x}{y},\mu^2) + P_{q\gamma}(y)\; e_i^2\gamma(\frac{x}{y},\mu^2)\Big\} \\
\frac{\partial g(x,\mu^2)}{\partial \log\mu^2} &= \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\Big\{P_{gq}(y)\; \sum_j q_j(\frac{x}{y},\mu^2) + P_{gg}(y)\; g(\frac{x}{y},\mu^2)\Big\} \\
\frac{\partial \gamma(x,\mu^2)}{\partial \log\mu^2} &= \frac{\alpha}{2\pi}\int_x^1 \frac{dy}{y}\Big\{P_{\gamma q}(y)\; \sum_j e_j^2\; q_j(\frac{x}{y},\mu^2) + P_{\gamma\gamma}(y)\; \gamma(\frac{x}{y},\mu^2)\Big\}\;, \qquad (1)
\end{aligned}$$

large logarithmic $\alpha\log(Q^2/m^2)$ contributions arise from photons emitted off incoming quark lines, the analogue of the $\alpha\log(Q^2/m_e^2)$ initial-state radiation corrections familiar in e^+e^- collisions. One could take these explicitly into account, but this would require a consistent choice of input quark masses. Furthermore, at the very high Q^2 scales probed at hadron colliders, one should in principle resum these logarithms. Fortunately the QCD factorisation theorem applies also to QED corrections, and as a result such collinear (photon-induced) logarithms can be absorbed into the parton distributions functions, exactly as for the collinear $\alpha_S \log Q^2$ logarithms of pQCD. There are two effects of this: the normal DGLAP evolution equations are slightly modified — the emitted photon carries away some of the quark's momentum — and a 'photon parton distribution' of the proton, $\gamma^p(x,Q^2)$, is generated. By correctly taking account of these QED effects through modified DGLAP evolution equations, which at leading order in α_S and α are shown listed Eq. (1) where the splitting functions are $\tilde{P}_{qq} = C_F^{-1}P_{qq}$, $P_{\gamma q} = C_F^{-1}P_{gq}$, $P_{q\gamma} = T_R^{-1}P_{qg}$ and $P_{\gamma\gamma} = -\frac{2}{3}\sum_i e_i^2\delta(1-y)$, we obtain a consistent procedure for dealing with this part of the overall electroweak correction in all hard-scattering processes involving initial-state hadrons[16]).

With the QED–modified DGLAP formalism, it is in principle straightforward to repeat the global NLO or NNLO (in pQCD) fit. However there is a complication because now we must allow for isospin symmetry breaking in all the distributions, that is $\gamma^p \neq \gamma^n \Rightarrow q^p \neq q^n \Rightarrow g^p \neq g^n$. This makes the evolution and fitting significantly more complex, and potentially more than doubles the number of parameters in the fit, a signficant fraction of which will not be at all well determined. We have performed fits at both NLO and NNLO, using the same input data as in the standard fit—for full details see Ref. [17]. In both cases the QED corrections do not alter the fit quality in any significant way. The resulting parton distributions for the proton are shown in Fig. 2. The quark and gluon distributions are very similar to the standard MRST parton distributions, but it is interesting to note the features of the new photon distribution. At $Q^2 = 20$ GeV^2 it is larger than the b-quark distribution, and also larger than the sea quarks at the highest values of x. The distributions for the neutron are similar, apart from slight differences in the valence quarks (we find $u^p < d^n$ at high x with the opposite behaviour at small x) and a smaller photon distribution. The isospin violation induced by the QED corrections has important implications for the anomaly in the measurement of $\sin^2\theta_W$ reported by the NuTeV collaboration[18]. The quantity measured by NuTeV is essentially the Paschos-Wolfenstein ratio

$$R^- = \frac{\sigma^\nu_{\rm NC} - \sigma^{\bar\nu}_{\rm NC}}{\sigma^\nu_{\rm CC} - \sigma^{\bar\nu}_{\rm CC}} \approx \frac{1}{2} - \sin^2\theta_W \qquad (2)$$

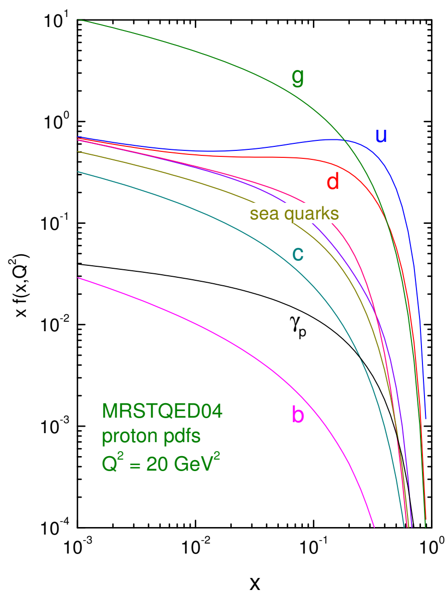

Figure 2. Proton parton distributions from a global fit containing $O(\alpha)$ QED corrections.[17].

where the second (approximate) equality assumes isospin symmetry, i.e. $u^p = d^n$ and $d^p = u^n$. If this not the case, as in our QED-improved fit, then the right-hand side has an extra contribution proportional to the first moment of the valence quark differences. Using our new distributions, we obtain a change in the measured value of $\sin^2\theta_W$ of -0.002, which corresponds to a little more than 1σ of the total NuTeV discrepancy. These results are in remarkable agreement with our previous analysis[11] of isospin-violating effects in parton distributions based on the Lagrange Multiplier method. There we found a shift of $\delta R^-_{\rm iso} = -0.002$, with 90% confidence level limits of $-0.007 < \delta R^-_{\rm iso} < +0.007$, comfortably more than needed to explain the NuTeV anomaly.

References

1. M. Botje, Eur. Phys. J. **C14** (2000) 285.
2. W.T. Giele and S. Keller, Phys. Rev. **D58** (1998) 094023; W.T. Giele, S. Keller and D.A. Kosower, hep-ph/0104052.
3. S.I. Alekhin, Phys. Rev. **D68** (2003) 014002.
4. CTEQ Collaboration: D. Stump *et al.*, Phys. Rev. **D65** (2002) 014012.
5. CTEQ Collaboration: J. Pumplin *et al.*, Phys. Rev. **D65** (2002) 014013.
6. CTEQ Collaboration: J. Pumplin *et al.*, JHEP 0207:012 (2002).
7. H1 Collaboration: C. Adloff *et al.*, Eur. Phys. J. **C21** (2001) 33.
8. H1 Collaboration: C. Adloff *et al.*, hep-ex/0304003.
9. A.M. Cooper-Sarkar, hep-ph/0205153, J. Phys. **G28** (2002) 2669; ZEUS Collaboration: S. Chekanov *et al.*, Phys. Rev. **D67** (2003) 012007.
10. A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, Eur. Phys. J. **C28** (2003) 455.
11. A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, Eur. Phys. J. **C35** (2004) 325.
12. S. Moch, J.A.M. Vermaseren and A. Vogt, Nucl. Phys. **B688** (2004) 101; **B691** (2004) 129.
13. U. Baur, S. Keller and D. Wackeroth, Phys. Rev. **D59** (1999) 013002; S. Dittmaier and M. Krämer, Phys. Rev. **D65** (2002) 073007; U. Baur and D. Wackeroth, hep-ph/0405191.
14. U. Baur *et al.*, Phys. Rev. **D65** (2002) 033007.
15. M. L. Ciccolini *et al.*, Phys. Rev. **D68** (2003) 073003.
16. A. De Rujula, R. Petronzio and A. Savoy-Navarro, Nucl. Phys. **B154** (1979) 394; J. Kripfganz and H. Perlt, Zeit. Phys. **C41** (1988) 319; J. Blümlein, Zeit. Phys. **C47** (1990) 89.
17. A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, hep-ph/0411040.
18. G.P. Zeller *et al.*, Phys. Rev. Lett. **88** (2002) 091802.