

# THE BNL G-2 EXPERIMENT: PRESENT LIMITS AND FUTURE PROSPECTS

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The anomalous magnetic moment of the muon has been measured to 0.5 ppm in a series of precision experiments at the Brookhaven Alternating Gradient Synchrotron. The individual results for each polarity:  $a_{\mu}^{+} = 11\,659\,204(7)(5) \times 10^{-10}$  and  $a_{\mu}^{-} = 11\,659\,214(8)(3) \times 10^{-10}$  are consistent with each other, so that we can write the average anomaly as  $a_{\mu}(\text{exp}) = 11\,659\,208(6) \times 10^{-10}$  (0.5 ppm). A discrepancy,  $\Delta a_{\mu}$ , between the measured value  $a_{\mu}(\text{exp})$  and the Standard Model  $a_{\mu}(\text{SM})$  would be a signal for new physics. Currently the standard model prediction is calculated to 0.6 ppm precision and is dominated by the uncertainty of the hadronic contributions. We expect that the error on  $a_{\mu}(\text{SM})$  will be reduced by a factor of two within the next decade, which, when combined with another experimental run at BNL, would put serious constraints on dark matter and supersymmetry.

## 1. Introduction

The magnetic moment is defined as  $\mu = g(e\hbar/2mc)\vec{s}/2$ , where  $g$  is the gyromagnetic ratio. Deviations from a purely pointlike  $g=2$  Dirac particle are characterized by the anomaly  $a = (g-2)/2$ . The anomaly for leptons is  $\sim 10^{-3}$  due to interactions with virtual particles which couple to the electromagnetic field, thus providing a laboratory for testing the Standard Model. Whereas the electron anomaly provides the most precise measurement of the fine structure constant  $\alpha$ , the muon anomaly is more sensitive by  $m_{\mu}^2/m_X^2$  to virtual  $W$  and  $Z$  gauge bosons, as well as any other, as yet unobserved, particles in the hundreds of GeV mass range.

## 2. Experimental Status of $a_{\mu}$

Pions produced on a nickel target are directed down a beamline which momentum selects the forward-going decay muons to produce a 96% polarized muon beam. The muons are injected into the magnet through a superconducting inflector magnet. A pulsed magnetic kicker bumps the muons onto stored orbits in a uniform 1.45 T field and electrostatic quadrupoles provide vertical focusing. The spin vector of the polarized muons precesses relative to the momentum vector with a frequency given by:

$$\omega_a = \omega_s - \omega_c = a_{\mu} \frac{eB}{mc} + \frac{e}{mc} \left( a_{\mu} - \frac{1}{\gamma^2 - 1} \right) \beta \times E$$

The dependence of  $\omega_a$  on  $E$  is eliminated to first order by choosing a  $\gamma$  which cancels out the second term in equation 1, corresponding to a muon momentum of  $p = 3.094$  GeV/c. The  $a_{\mu}$  is then extracted from the ratio of the measured anomalous precession  $\omega_a$  to the free proton precession frequency  $\omega_p = \mu_p B / \hbar$  in the same magnetic field. The proton magnetic moment enters as the ratio  $\lambda = \mu_{\mu} / \mu_p$  measured by the muonium hyperfine structure interval [1].  $B$  is measured in situ every few days by a trolley with 17 NMR probes, and interpolated between trolley runs using  $\sim 150$  stationary probes.

To find  $\omega_a$ , the decay positrons (electrons) from  $\mu^{+(-)} \rightarrow e^{+(-)} \nu \bar{\nu}$  are detected by 24 lead-scintillating fiber calorimeters read out by 400 MHz waveform digitizers, yielding both time and energy information. Since this is a weak decay, the high energy positrons preferentially point in the direction of the muon spin, such that an energy threshold cut at 1.8 GeV produces a modulation in the number of positrons detected as a function of time, multiplied by the muon decay curve:

$$N(t) = N_0 e^{-t/\gamma\tau} (1 + A \cos(\omega_a t + \phi))$$

where  $A$  (asymmetry) is the depth of the modulation and  $\tau$  is the muon lifetime at rest.

This form is modified by beam dynamics, pileup, gain corrections at early times, and muon losses coming from processes other than decay. Differences in the way in which each of these effects is treated, as well as data selection and pulse finding, resulted in 4 independent analyses of  $\omega_a$  for the 2000 data [2] and 5 for the 2001 data [3], which are then averaged, with attention to their correlated uncertainties.

The analysis of  $\omega_a$  and  $\omega_p$  is divided into separate tasks with secret offsets for self-blinding. The value of  $a_\mu$  is determined after the analyses of  $\omega_p$  and  $\omega_a$  have been finalized, the offsets removed, and radial E-field and pitch corrections applied. Assuming CPT, we combine the results from  $\mu^+$  and  $\mu^-$  runs to obtain  $a_\mu(\text{exp}) = 11\,659\,208(6) \times 10^{-10}$  [3]

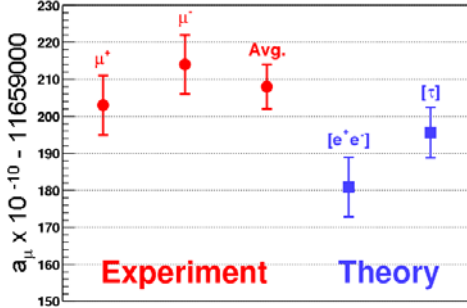


Figure 1. Comparison of  $a_\mu(\text{exp})$  from the  $\mu^+$  (2000) and  $\mu^-$  (2001) BNL runs with  $a_\mu(\text{SM})$  using  $a_\mu(\text{Had})$  from both  $e^+e^-$  and  $\tau$ -decay parameterizations.

### 3. Theoretical Status of $a_\mu$

The difference:  $\Delta a_\mu = a_\mu(\text{exp}) - a_\mu(\text{SM})$  reveals new physics, where we can write  $a_\mu(\text{SM}) \times 10^{10}$  as:

$$a_\mu(\text{QED}) = 1111658471.93 \quad (0.12) \quad [4]$$

$$a_\mu(\text{Weak}) = 15.4 \quad (0.2) \quad [5]$$

$$a_\mu(\text{Had-LO}) = 693.4 \quad (5.3)(3.5) \quad [6]$$

$$a_\mu(\text{Had-NL}) = -9.8 \quad (0.1) \quad [6]$$

$$a_\mu(\text{Had l-b-l}) = 13.6 \quad (2.5) \quad [7]$$

representing the latest compilations at the time of this conference. The QED component dominates, but also has the smallest error (now computed up  $\alpha^4$ , with an estimation of  $\alpha^5$ ). The weak contribution includes 2-loop, leading and next-leading log, but hasn't changed much in the last decade. The largest error is in the hadronic vacuum polarization contributions which cannot be calculated from perturbative QCD, but instead must be related to the measured hadron production cross section  $R(s)$  in  $e^+e^-$  collisions via a dispersion relation. Calculations of  $a_\mu(\text{Had})$  using vector spectral functions from hadronic  $\tau$ -decays [8] gives a contribution that differs significantly from the  $e^+e^-$  determination dominated by Novosibirsk CMD-2 data, as well as from recent KLOE [9] and BaBar results which use radiative return to reduce the center of mass energies to those most relevant to  $g-2$ . It is no longer used in direct comparisons as it requires assumptions about CVC, isospin corrections, electroweak symmetry breaking, and the charged- $\rho$  mass [10]. The next largest error is the light-by-light term which is a model-dependent calculation. Using the numbers quoted above, the  $g-2$  discrepancy is  $\Delta a_\mu = 23.5 \quad (9.0) \times 10^{-10}$  representing a  $2.6\sigma$  significance.

### 4. Constraints on SUSY

If supersymmetry is responsible for the non-standard part of the  $g-2$  anomaly, there exist new one-loop diagrams which can contribute to  $a_\mu$ . For minimal supersymmetry in the limit of large  $\tan\beta$ , the chargino contribution can most easily generate masses large enough to explain the discrepancy. A fairly generic result [11] for  $\tan\beta > 5$  is an inverse quadratic dependence on the SUSY loop mass given by

$$|a_\mu^{\text{SUSY}}| \cong 13 \times 10^{-10} \left( \frac{100 \text{ GeV}}{m_{\text{SUSY}}} \right)^2 \tan\beta$$

where  $\tan\beta$  is the ratio of vacuum expectation values of the Higgs doublet. Thus, a large deviation from SM constrains  $m_{\text{SUSY}}$  to lower

masses. Conversely, if the LHC eventually measures  $m_{\text{SUSY}}$ , then  $\Delta a_\mu$  will constrain  $\tan\beta$ . When such constraints are translated into a 2-D plot of gaugino ( $m_{1/2}$ ) vs slepton ( $m_0$ ) mass in constrained minimal SUSY [12], they form the quarter circle shape of the g-2 preferred mass

the dispersion integral to improve as that work is completed. An upgrade to the VEPP-2000 collider and an intensity upgrade at the BEPS machine will increase the sample of e+e- data in low to intermediate energy ranges. BaBar, KLOE and Belle will weigh in on differential

Table 1. Evolution of the uncertainties in the BNL g-2 experiment, including estimated errors on the proposed experiment.

Data Set:	1997	1998	1999	2000	2001	2009
<b>Comments</b>	$\pi$ -injection Engineering Run	kicker installed field stabilized	1 <sup>st</sup> long run	new inflector	Reverse polarity	New BNL E969 21 week run
<b>Statistics</b> ( $N_e$ above $E_{\text{thr}}$ )	<b>12.5 ppm</b> 12 M e <sup>+</sup>	<b>4.9 ppm</b> 84 M e <sup>+</sup>	<b>1.25 ppm</b> 1 B e <sup>+</sup>	<b>0.6 ppm</b> 4 B e <sup>+</sup>	<b>0.7 ppm</b> 4 B e <sup>-</sup>	<b>0.14 ppm</b> 70 B e <sup>+</sup>
<b>Systematics</b>	<b>2.9 ppm</b>	<b>1 ppm</b>	<b>0.5 ppm</b>	<b>0.4 ppm</b>	<b>0.3 ppm</b>	<b>0.15 ppm</b>
<b><math>\delta\omega_a</math></b>	2.6 ppm	0.7 ppm	0.3 ppm	0.3 ppm	0.21 ppm	0.1 ppm
<b>Dominated by</b>	WFD threshold pion flash	pileup AGS mistune	pileup AGS mistune	coherent betatron $\mu$ loss, pileup	gain stability $\mu$ loss	
<b><math>\delta\omega_p</math></b>	1.3 ppm	0.5 ppm	0.4 ppm	0.24 ppm	0.17 ppm	0.1 ppm
<b>Dominated by</b>	thermal fluctuations no active feedback	trolley position inflector	trolley position inflector	trolley position	trolley position	trolley position

region in Figure 2. The dotted lines represent the 1- $\sigma$  contours and the solid lines bounding the shaded region correspond to the 2- $\sigma$  contours on a g-2 discrepancy presumed to be saturated by the SUSY contribution. As  $\tan\beta$  is increased, the quarter circle stretches and moves to higher mass. Both the positive nature of the g-2 discrepancy and the  $b \rightarrow s\gamma$  branching ratio constraint prefer positive  $\mu$ . The power of the g-2 measurement to constrain SUSY dark matter lies in the contrasting way in which it cuts across  $m_0 - m_{1/2}$  parameter space compared to the cosmologically preferred region, which is the hyperbolically thin line with co-annihilation strips extending to high  $m_{1/2}$  and  $m_0$ .

#### 4. Future Expectations

Since only 20% of the CMD-2 e+e- data (center of mass energies from 0.3–1.4 GeV) has been analyzed so far, we can expect the precision in

cross sections using radiative return for multiple pion states. Within the decade, the error on the 1<sup>st</sup> order hadronic correction should be reduced to  $\delta a_\mu \sim 35 \times 10^{-11}$  which is comparable to the uncertainty on the hadronic light-by-light contribution.

On the experimental side, Table 1 shows the evolution of the measured g-2 precision. It can be seen that each run is statistics limited and that the systematic uncertainties for  $\omega_a$  and  $\omega_p$  are comparable. Another run in 2009 as E969 will represent the best one can do with the modified ring and detector geometry, before becoming limited by systematics. This requires collecting 70 billion decay positrons. By doubling the number of beamline quadrupoles and using an open-ended inflector design, the number of stored muons can be increased by a factor of 5, allowing this to be done in only 21 weeks for a 0.14 ppm statistical error. The systematic error on  $\omega_a$  can be reduced by

injecting backward-going muons to reduce pion flash, adding another kicker module to reduce coherent betatron oscillations, segmenting calorimeters to reduce rate-dependent effects, and improving the front end electronics and data acquisition to handle the increased throughput. In situ measurement of the kicker eddy currents and mapping of the NMR probes can reduce  $\delta\omega_p$ . Combined with the theory precision expected a few years from now, the error on  $\Delta a_\mu$  would then be at  $4.7 \times 10^{-10}$ . If the mean  $\Delta a_\mu$  remains stable, this represents a  $6\sigma$  departure from the Standard Model.

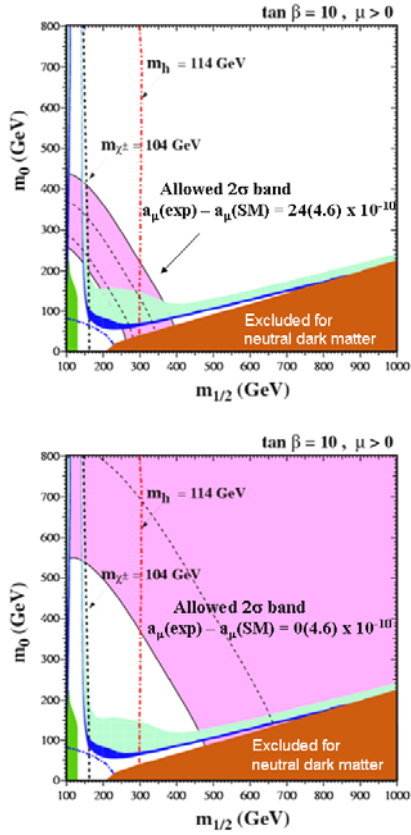


Figure 2. Courtesy of Keith Olive. The  $m_{1/2}$  vs  $m_0$  planes in CMSSM. The cosmologically-preferred region allowed by WMAP constraint ( $0.094 < \Omega_{\text{CDM}} h^2 < 0.129$ ) is the thinner dark boomerang. The shaded region is favored by  $g-2$  at the  $1\sigma$  level (dotted lines) and  $2\sigma$  level (solid lines).

Figure 2 shows how this translates into dark matter constraints for a particular choice of

$\tan\beta=10$  and the preferred  $\mu>0$ . Both plots include the factor of 2.5 reduction in uncertainty expected from a new run at BNL, combined with improvements in  $a_\mu(\text{Had})$  from  $e^+e^-$  data already collected. The plot at the top represents the case where the mean discrepancy remains stable at its present value. The plot below represents the case where the mean shifts down to the SM value. Due to the nature of the constraints, a reduction in the error bars which leaves the mean  $\Delta a_\mu$  intact will significantly narrow the band of allowed masses, while a shift down to SM will widen the allowed region, but reject SUSY masses  $< 500 \text{ GeV}/c^2$ .

#### 4. Conclusions

The popularity of SUSY as an answer to the hierarchy problem and as a means to unify gauge couplings has renewed interest in  $a_\mu$ , since the hint of a discrepancy points to such convenient SUSY masses. The experimental result has lead theorists to improve the hadronic vacuum polarization and higher order QED terms, uncover errors, spurred further experimental work on  $R(s)$ , and lead to a re-examination of CVC and pion form factors. Thus, no matter what the current theoretical fad, precision measurements of fundamental constants are an enduring contribution to physics, since they confront our preconceptions with reality and guide future discussions.

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